HELSINKI INSTITUTE OF PHYSICS INTERNAL REPORT SERIES

HIP-2012-03

Holographic Models for Large-N Gauge Theories

Janne Alanen

Division of Elementary Particle Physics
Department of Physics
Faculty of Science
University of Helsinki
Helsinki, Finland

ACADEMIC DISSERTATION

To be presented, with the permission of the Faculty of Science of the University of Helsinki, for public criticism in the auditorium E204 at Physicum, Gustaf Hällströmin katu 2 B, Helsinki, on August 14th 2012, at 12 o'clock.

Helsinki 2012

ISBN 978-952-10-5339-9 (printed version)
ISSN 1455-0563
ISBN 978-952-10-5340-5 (pdf version)
http://ethesis.helsinki.fi
Unigrafia
Helsinki 2012

J. Alanen: Holographic Models for Large-N Gauge Theories,
University of Helsinki, 2012, 59 pages,
HIP INTERNAL REPORT SERIES, HIP-2012-03
ISSN 1455-0563
ISBN 978-952-10-5339-9 (printed version)
ISBN 978-952-10-5340-5 (pdf version)

Keywords: Holography, AdS/CFT, gauge/gravity duality, IHQCD, quasi-conformal theory, string theory

Abstract

Gauge theories are used to describe interactions between the elementary particles of the standard model and beyond standard model theories. In the regime where interactions are strong perturbative methods cannot be used. Thus the non-perturbative part is generally studied by using lattice simulations and effective field theories. However a new method for exploring the non-perturbative part is the AdS/CFT duality that relates a specific string theory and a conformal field theory. In this thesis, the AdS/CFT duality is generalized to non-conformal gauge theories and its implications are studied. In particular, a holographic model for studying various large-N gauge theories is introduced.

Acknowledgements

This thesis is based on research carried out in Helsinki Institute of Physics and the Department of Physics of University of Helsinki. This work was made possible by the Magnus Ehrnrooth Foundation.

I wish to thank my supervisors Esko Keski-Vakkuri and Keijo Kajantie. They have taught me a lot about research, physics and life in general. Esko's deep knowledge on various fields has inspired me during graduate studies. Keijo, with his way of understanding physics behind the equations, is simply admirable. It has been a pleasure to work and learn from these wonderful friends. I am also grateful to my collaborators Ville Suur-Uski, Kimmo Tuominen, Timo Alho and Per Kraus.

I wish to thank the pre-examiners of this thesis, Aleksi Vuorinen and Tuomas Lappi for careful reading of the manuscript. Also Samu Kurki and Ville Keränen deserve thanks for reading and commenting the manuscript.

My studies at the University of Helsinki have been blessed by many wonderful and very sharp friends including both Villes, Matti, Olli and Samu. I really feel that I have had a unique opportunity to learn from the brightest young minds the world has to offer. Debates on life, science and in the badminton courts with Ville Suur-Uski have teached me many wonderful lessons. Ville is a person whom I admire in every part of life.

I feel lucky to have lovely fellows close to me with whom I have shared my life. A big part of my heart belongs to you, so thank you Villes, Esko, Micke, Tero, Otto, Maiju, Jussi, Jari, Markus, Eeva-Kaisa, Antti and all you out there.

Finally, I am deeply grateful to my family for supporting me in every aspect of my life. Words are not enough to express how much I love you.

Helsinki, June 2012

Janne Alanen

Contents

List of included papers

The publications included in this thesis are:

I. J. Alanen, K. Kajantie and V. Suur-Uski
"*A gauge/gravity duality model for gauge theory thermodynamics*"
Phys. Rev. D **80**, 126008 (2009), arXiv:0911.2114 [hep-ph]

II. J. Alanen and K. Kajantie
"*Thermodynamics of a field theory with infrared fixed point from gauge/gravity duality*"
Phys. Rev. D **81**, 046003 (2010), arXiv:0912.4128 [hep-ph]

III. J. Alanen, K. Kajantie and K. Tuominen
"*Thermodynamics of Quasi Conformal Theories From Gauge/Gravity Duality*"
Phys. Rev. D **82**, 055024 (2010), arXiv:1003.5499 [hep-ph]

IV. J. Alanen, T. Alho, K. Kajantie and K. Tuominen
"*Mass spectrum and thermodynamics of quasi-conformal gauge theories from gauge/gravity duality*"
Phys. Rev. D **84**, 086007 (2011), arXiv:1107.3362 [hep-th]

The author's contribution to the joint publications

In the first paper the author did some of the analytic and numerical calculations. The author constructed the beta functions that were used in calculations and the results were analyzed jointly with the collaborators.

In the second paper the author came up with the idea of studying the thermodynamics of fixed point gauge theory. Analytic and numerical calculations were done jointly with the collaborator. The result were analyzed together with collaborator.

In the third and fourth paper, the author contributed to the idea that the quasi-conformal gauge theories could be studied by using the IHQCD model. The author came up with a method for constructing the potential that was used in the calculations. The numerical code was partly developed by the author. The author did some of the analytic and numerical calculations jointly with the collaborators. The result were analyzed together with the collaborators.

Chapter 1

Introduction

The interactions of the standard model of particle physics can be described by gauge theories. The gauge group of the standard model is $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ and the interactions between elementary particles like electrons and quarks are mediated by gauge bosons. The gauge bosons transform under the representations of the standard model gauge group.

The $\mathrm{SU}(3)$ part describes interactions between quarks and the gauge bosons (the gluons). It is known as quantum chromodynamics (QCD) discovered in 1972. Since $\mathrm{SU}(3)$ is a non-Abelian gauge group, the gauge bosons carry (color-)charge similar to the color charged quarks. This non-Abelian structure implies that, even without quarks, the dynamics of pure gauge theory is very interesting and challenging. Further, QCD is strongly coupled in the infrared (IR) and, thus, the standard method for solving gauge field theories, i.e., perturbation theory cannot be used to study it at low energies. Due to the non-Abelian structure and strong coupling, QCD is hard to work with and still, after more than forty years, many phenomena lack detailed explanation. For example. although it was first introduced to explain some of the properties of low energy excitations, which are the hadrons seen by particle detectors, explaining these from first principles is demanding. Luckily, the flow of the QCD coupling makes the coupling weaker in the UV and, at very large energies, QCD behaves as a free theory, i.e., it is asymptotically free. Although particle detectors can only see the low energy excitations of QCD, asymptotic freedom makes it possible to use perturbation theory at the point where energy density is very high. This condition is satisfied right after the collisions in hadron colliders (RHIC and LHC) and the outcome of the experiments is affected by the fundamental interactions between the high energy excitations i.e. quarks and gluons. This is how one makes predictions and verifies that the hadrons and mesons are made up by the quarks and the gluons whose dynamics are covered by QCD.

The problems with QCD can be more or less generalized to other non-Abelian gauge theories, for example, to technicolor theories which may play a role beyond the standard model physics. Thus, studying the non-perturbative part of QCD, or more generally Yang-Mills theory with gauge group $\mathrm{SU}(N)$, is crucial for understanding the standard and beyond the standard model physics. The non-perturbative part of QCD (or $\mathrm{SU}(N)$) can be studied by different kinds of methods. One of the most powerful of these is the lattice QCD [1]. In lattice QCD, spacetime is discretized and one uses computers to calculate various quantities in the regime where perturbative calculations are not valid. Another way to attack strongly coupled QCD is to use different types of effective theories. One example of such a theory is chiral perturbation theory [2], where the dominant degrees of freedom of QCD at low energies are small-mass Goldstone bosons. In addition to these traditional methods, there is also quite a new way to study non-perturbative QCD: AdS/CFT duality [3]. This connection between gauge theory and string theory in Anti de Sitter (AdS) spacetime is the main subject of this thesis and is made more precise in the following sections.

In the late sixties, string theory originated from attempts to understand the strong interaction [4, 5]. By using string theory, some properties of hadrons can be explained easily compared to QCD. For example, the linear dependence of the mass (squared) as a function of their spin, i.e., the Regge trajectories [6], is easily explained by string theory. However, the quantization of string theory leads to particles that have not been seen in experiments, namely massless spin-2 particles. The massless spin-2 particle predicted by string theory was recognized as a graviton i.e. the quantum of gravity interaction and this shifted string theory from being a theory for strong interactions, to be a "theory of everything" which unifies all interactions including gauge theories and Einsteins gravity.

After the seventies, QCD and string theory seemed to separate into two different frameworks which did not have that much common. Then, in 1997, Juan Maldacena found surprising and a far reaching connection between gauge and string theory [3]. He conjectured a duality between the supersymmetric $\mathcal{N} = 4$ SU(N) gauge theory ($\mathcal{N} = 4$ SYM) in $3 + 1$ dimensions and type IIB superstring theory on $AdS_5 \times \mathrm{S}^5$. This conjecture is known as AdS/CFT duality, where CFT (conformal field theory) is the supersymmetric $\mathcal{N} = 4$ SU(N) gauge theory which is conformally invariant, i.e., has a vanishing beta function. The duality was made more precis by Witten [7] and Klebanov, Gubser, Polyakov [8]. Further, this conjecture has been confirmed by highly non-trivial tests and so far no exception has been found. This remarkable duality between gauge theory and the string theory in a certain background is non-trivial and can also be thought as a first realization of the holographic principle introduced by 't Hooft [9] and Susskind [10] in the early nineties. The holographic principle is thought to be a property of quantum gravity and it states that the degrees of freedom of gravitating system bound to some $D + 1$ volume can be rewritten as a theory acting on the D dimensional boundary of the volume. One of its implications is that the information on the black hole should somehow be encoded into its event horizon. Thus, it gives an explanation for the black hole entropy formula found by Bekenstein and Hawking.

Assuming that the AdS/CFT conjecture is true and exact, it can be viewed as a non-perturbative definition of the quantum gravity (the string theory), since the non-perturbative quantum mechanical definition of the $\mathcal{N} = 4$ SYM is known and can be written as a path integral.

It is interesting to study different limits of the duality. The $\mathcal{N} = 4$ SU(N) gauge theory has two parameters; the rank of the gauge group N (or the number of the degrees of freedom) and the 't Hooft coupling $\lambda = g^2_{\mathrm{YM}} N$ [11]. The duality relates the rank of the gauge group to quantum effects in string theory. In more detail, in the limit $N \to \infty$ the quantum effects of string theory are suppressed and, thus, it reduces to its classical limit. The strong 't Hooft coupling limit ($\lambda \to \infty$) of the gauge theory corresponds to string theory with the fundamental string length taken to zero ($l_{\mathrm{string}} \to 0$). Hence in this limit string theory reduces to (super)gravity. Altogether, taking the double limit $N \to \infty$ and $\lambda \to \infty$ in the gauge theory corresponds to the classical (super)gravity approximation of string theory. This limit in the gauge theory is non-perturbative and is usually thought to be very hard to solve as explained above. Now, using the duality, the non-perturbative regime is related to classical (super)gravity. Studies related to conjecture have been very active ever since it was introduced and the conjecture is further explored in this thesis.

There are some key differences between $\mathcal{N} = 4$ supersymmetric SU(N) gauge theory and non-supersymmetric gauge theories, like SU(3) QCD or Technicolor theories [12, 13, 14]. First, the $\mathcal{N} = 4$ SYM is a conformal field theory implying that its beta functions vanishes. The conformality of the theory is a huge simplification, since some gauge invariant operators are independent of the energy scale at which they are studied, which is in strong contradiction compared to QCD. One important example of phenomena seen in nature, is color confinement, which implies that the low energy excitations of QCD are not massless quarks and gluons, but massive hadrons with dynamically generated mass scale $\Lambda_{\rm QCD}$. This type of dynamically generated scale cannot be seen in conformally invariant $\mathcal{N} = 4$ SYM and, thus, the low energy excitations of the theory stay massless. Secondly, $\mathcal{N} = 4$ SYM contains quarks which are superpartners of the gluon fields and, so, they transform as the adjoint representation of SU(N). This is different from what is seen in nature, where the quarks transform under the fundamental representation of SU(N). In technicolor theories, the technifermions may transform as adjoints of the gauge group and, thus, can be more close to $\mathcal{N} = 4$ SYM than QCD [15]. A third problem is related to the regime, where the conjecture and the relations between the theories are well known. This is the classical string theory approximation, which is valid when the number of gluonic fields is large (or $N \to \infty$). In nature, the rank of the QCD gauge group is three ($N = 3$) which it is not quite infinite and the classical supergravity approximation may not be valid for theories seen in nature. Luckily, there are studies that point in the direction that, at least some of the properties of the gauge theory, are insensitive to the rank of the gauge group when it is greater or equal to three ($N \geq 3$) [16].

After all of these differences between $\mathcal{N} = 4$ SYM and non-supersymmetric gauge theories, one would thing that using the AdS/CFT duality to solve, for example the mass spectrum of QCD mesons, would not be justified (there are not even QCD type massive mesons in SYM). Fortunately, there still is hope, since slight modifications of AdS geometry will correspond to different types of gauge theories that are not that far from those seen in nature. One obvious thing to do is to study a gauge theory at finite temperature [7]. Finite temperature breaks supersymmetry and what is left is quite close to non-supersymmetric SU(N) Yang-Mills theory. Within the limit in which string theory reduces to classical gravity, adding finite temperature to field theory corresponds to adding a black hole with Hawking temperature $T_{\rm BH}$ in to the AdS background. Thus, one has connection between a strongly coupled finite temperature SU(N) gauge theory and a black hole in an asymptotically AdS space. Further, this regime of the field theory is close to what is studied using hadron colliders, which can produce a phase of the matter called quark-gluon plasma [17]. Interestingly, by using the duality, one may get some hint about phenomena taking place in the collider by studying classical black holes in asymptotically AdS spacetime.

Another way to get dual models for more realistic field theories is to add something similar to the QCD scale, $\Lambda_{\rm QCD}$, to the gravity setup. There are many different ways to do this, but one of the simplest ways is to add a cut-off to AdS space (see [18]) that is dual to gauge theory with a mass scale and, thus, is closer to SU(N) QCD. These kinds of models are called bottom-up approaches and generally go under the name of an AdS/QCD duality (or a gauge/gravity duality). A review of these methods can be found in [19] and references therein. There are also so called top-down models, which lie on a slightly more solid ground than bottom-up approaches. In the top-down models [20, 21], the starting point is usually some known connection between a gauge and a string theory in higher dimensional spacetime. Further, the compactification of string theory to some compact manifold gives rise to a mass scale and leads to broken low energy supersymmetry. Thus one has exact duality between four dimensional QCD type of theory and the gravity setup.

The model used in this thesis is called Improved Holographic QCD (IHQCD) [22, 23, 24], which can be thought to lie somewhere between the top-down and the bottom-up models. The starting point of the model is a five dimensional (non-critical) string theory, where the running of the QCD 't Hooft coupling is taken care of by a dilaton field. This model dynamically generates a scale dual to the QCD mass scale $\Lambda_{\rm QCD}$ and also the phase transition of QCD has its counterpart. The bottom-up ingredients are connected to a choice of the dilaton potential, so that the phenomena seen in nature or in lattice QCD can also be seen using this dual picture. The problems of the model include the hostile environment of the non-critical string theories which usually generate curvatures close to inverse string length. This leads to a conclusion that the use of a two derivative action for gravity may not be justified. Still, this method can hopefully be used to give at least a qualitative picture of gauge theory phenomena and to calculate quantities that are very hard to find by using traditional methods of QCD.

1.1 Organization of the thesis

The thesis consists of four articles and of an introductory part, divided in four Chapters. The introductory part is intended as an overview of some of the essential tools for studying strongly interacting field theories using dualities between gauge theories and string theory.

In the second Chapter of the introductory part, the Maldacena conjecture is introduced and its use to study field theory phenomena in the language of classical gravity is discussed. Further, some extensions and modifications of the original duality are introduced.

In the third Chapter, the model used in four articles is introduced and some of the results of [22, 23, 24] are reviewed. More precisely, the vacuum and the black hole solutions are studied and criteria for a confined/deconfined phase transition and for a mass gap are introduced. At the end of the Chapter, some comparison with lattice QCD data is also made. Finally, Chapter four is the summary.

The four articles provide the core part of this thesis. In the first paper some modifications of the IHQCD were presented, and analytic calculation of thermodynamics quantities were done. In the second article, a gravity dual to field theory with a fixed point in the infrared was studied and the phase structure of the theory was examined. In the third paper, a generalization to IHQCD was presented and it was used to study quasi-conformal theories. In the last article, the mass spectrum of the quasi-conformal theories was explored.

Chapter 2

Gauge/Gravity duality

The AdS/CFT duality was conjectured by Maldacena in 1997 [3] and has ever since been one of the most studied branches of string theory. This duality between a gauge and gravity theory is highly non-trivial and can be thought as a first realization of the holographic principle introduced by 't Hooft and Susskind in the early nineties [9, 10]. The holographic principle is conjectured to be a general property of quantum gravity and it states that the degrees of freedom (d.o.f) of a gravitating system bound to $D+1$ dimensional volume can be rewritten as gauge theory d.o.f living on D dimensional boundary. In the case of the AdS/CFT duality, the gravitating system is a string theory in an asymptotically Anti de Sitter (AdS) space and the gauge theory d.o.f are described by a conformal field theory on the boundary of the AdS space. More precisely, Maldacena conjectured an exact duality between type IIB superstring theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and $\mathcal{N} = 4$ superconformal $\mathrm{SU}(N)$ Yang-Mills theory in four dimensions.

String theory on curved manifolds is known to be a very challenging problem but one can instead study a special limit to this duality, which is the large 't Hooft limit of the gauge theory [11]. In this limit, the duality connects classical gravity (or classical supergravity) to strongly coupled field theory. In particular, within this limit, the AdS/CFT correspondence, or more generally the gauge/gravity duality introduces new tools for studying some fundamental problems of non-perturbative field theories. For example, the AdS/CFT duality can be used to study the behavior of the strongly coupled quark-gluon plasma found in hadron colliders like RHIC and recently in LHC [17].

The gauge/gravity duality can also be used to study phenomena linked to condensed matter physics. For example, there are interesting applications to quantum Hall effect, high temperature superconductivity and non-Fermi liquids. A recent review of these applications can be found in [25].

2.1 Maldacena's conjecture

Below we give a very brief introduction to Anti de Sitter space (AdS), conformal field theory (CFT), supersymmetry (SUSY), string theory and D-branes. As one can notice, every one of these subjects could be the topic of complete thesis and therefore we introduce only necessary tools for understanding the conjecture. After introducing these tools, we formulate the conjecture and study of its properties. A more complete introduction to these subjects can be found in [26, 27, 28, 29]. The discussion in this section follows closely the one made in [30].

2.1.1 Anti de Sitter space

Anti de Sitter space (AdS) is a maximally symmetric solution to Einstein equations with a negative cosmological constant. The Einstein-Hilbert action in $D+1$ dimension is

$$S = \frac{1}{16\pi G_{D+1}} \left(\int_M \mathrm{d}^{D+1}x\sqrt{-g}(R + \frac{D(D-1)}{\mathcal{L}^2}) - 2\int_{\partial M} \mathrm{d}^D x\sqrt{-\gamma}K \right), \tag{2.1}$$

where the second term is the Gibbons-Hawking term, which is relevant to spacetimes with a boundary ∂M. The induced metric on the boundary is $\gamma_{\mu\nu}$ and $K = K^{\mu\nu}\gamma_{\mu\nu}$ is the trace of the second fundamental form $K_{\mu\nu}$ and G_{D+1} is Newton's constant. The boundary term does not affect the equations of motion that are given by Einstein equations

$$R_{MN} - \frac{1}{2}Rg_{MN} - \frac{D(D-1)}{2\mathcal{L}^2}g_{MN} = 0. \tag{2.2}$$

A solution to these equations is anti de Sitter (AdS) space

$$\mathrm{d}s^2 = g_{MN}\mathrm{d}x^M\mathrm{d}x^N = \frac{\mathcal{L}^2}{z^2}\left(\mathrm{d}z^2 + \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu\right), \tag{2.3}$$

where $\eta_{\mu\nu}$ is the D dimensional Minkowskian metric. Symmetries of the AdS solution become more transparent if one considers an embedding of the $D+1$ dimensional AdS space to the $D+2$ Minkowskian spacetime with two timelike coordinates. The metric of the flat $D+2$ dimensional Minkowskian spacetime is

$$\mathrm{d}s^2_{D+2} = \mathrm{d}z^2 + \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu - \mathrm{d}\tau^2, \tag{2.4}$$

which clearly has a Lorentz symmetry $\mathrm{SO}(D,2)$ and a translation symmetry. The AdS embedding is given by the equation

$$\mathcal{L}^2 = z^2 + \eta_{\mu\nu}x^\mu x^\nu - \tau^2 \tag{2.5}$$

which is also invariant under $\mathrm{SO}(D,2)$ but breaks the translation invariance. Using this equation, one can eliminate $\mathrm{d}\tau$ from the $D+2$ Minkowskian metric (2.4) from which, by using specific coordinate transformations [30], one finds $D+1$ dimensional AdS metric similar to (2.3). The outcome of this analysis is that the global symmetry of the $D+1$ dimensional AdS space is $\mathrm{SO}(D,2)$. Furthermore, the group $\mathrm{SO}(D,2)$ has $(D+2)(D+1)/2$ generators which is equal to the maximal number of Killing vector in $D+1$ dimensional space and, thus, the AdS space is a maximally symmetric solution to Einstein equations. Other maximally symmetric solutions are the flat space ($\mathcal{L}^2 \to \infty$) and the de Sitter space dS^{D+1} ($\mathcal{L}^2 \to -\mathcal{L}^2$).

The metric (2.3) appears to be singular at $z \to 0$, but since the curvature invariants

$$R = -\frac{(D+1)D}{\mathcal{L}^2}, \quad R_{\mu\nu} = -\frac{D}{\mathcal{L}^2}g_{\mu\nu}, \quad R_{\mu\nu}R^{\mu\nu} = \frac{D^2(D+1)}{\mathcal{L}^4} \tag{2.6}$$

are regular, it can be identified as a coordinate singularity. Further, the $z \to 0$ regime can be identified as the boundary of the AdS space. The boundary metric is conformal to the $(D-1)+1$ dimensional flat Minkowskian metric. In the boundary, the symmetry transformations of the AdS space are equivalent to conformal symmetry transformations in the $(D-1)+1$ dimensional Minkowskian spacetime. The conformal group in $D+1$ dimensions is $\mathrm{SO}(D+1,2)$ and it generates symmetries that are the Poincaré symmetries, a scaling symmetry

$$x^\mu \to \lambda x^\mu \tag{2.7}$$

and a special conformal symmetry

$$x^\mu \to \frac{x^\mu - b^\mu x^2}{1 - 2x_\mu b^\mu - b^\mu x^2}. \tag{2.8}$$

Implications of the conformal symmetry to field theories are discussed in the subsection 2.1.3.

2.1.2 Anti de Sitter black holes

Black holes in asymptotically AdS are static and spherical symmetric solutions to Einstein equations (2.2) with a delta functional source of matter. The metric ansatz[1] for a static and spherically symmetric solution is

$$\mathrm{d}s^2 = -\mathcal{A}(r)\mathrm{d}t^2 + \mathcal{B}(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-1}, \tag{2.9}$$

which can be simplified by defining $\mathcal{B}(r) = 1/\mathcal{A}(r)$. The Einstein equations for the components R_{tt} and R_{rr} are[2]

$$\frac{1}{2}\mathcal{A}\frac{[2\mathcal{A}' + r\mathcal{A}'']}{r} = \frac{D}{\mathcal{L}^2}\mathcal{A}, \tag{2.10}$$

$$-\frac{1}{2}\frac{[2\mathcal{A}' + r\mathcal{A}'']}{\mathcal{A}r} = -\frac{D}{\mathcal{L}^2}\mathcal{A}^{-1}, \tag{2.11}$$

which both are solved by

$$\mathcal{A}(r) = \frac{r^2}{\mathcal{L}^2} + C_1 + \frac{C_2}{r^{D-2}}. \tag{2.12}$$

Thus, the black hole solution in asymptotically AdS space is

$$\mathrm{d}s^2 = -\left(\frac{r^2}{\mathcal{L}^2} + C_1 + \frac{C_2}{r^{D-2}}\right)\mathrm{d}t^2 + \left(\frac{r^2}{\mathcal{L}^2} + C_1 + \frac{C_2}{r^{D-2}}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-1}, \tag{2.13}$$

where the values for the coefficients C_i can be found by considering a limit where $\mathcal{L} \to \infty$, which corresponds to a spherical Schwarzschild solution in asymptotically flat space. The limit $\mathcal{L} \to \infty$ for the AdS black hole is

$$\mathrm{d}s^2_{\mathrm{AdS}} = -\left(C_1 + \frac{C_2}{r^{D-2}}\right)\mathrm{d}t^2 + \left(C_1 + \frac{C_2}{r^{D-2}}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-1}, \tag{2.14}$$

[1] In these coordinates the boundary of AdS space is at $r \to \infty$ ($r = \mathcal{L}^2/z$).

[2] Other components of the Riemann tensor are trivial.

and the spherical Schwarzschild solution in flat space is [30]

$$\mathrm{d}s^2_{\mathrm{Schw}} = -\left(1 - \frac{16\pi G_{D+1}}{(D-1)\Omega_{D-1}}\frac{1}{r^{D-2}}\right)\mathrm{d}t^2 + \left(1 - \frac{16\pi G_{D+1}}{(D-1)\Omega_{D-1}}\frac{1}{r^{D-2}}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-1}. \tag{2.15}$$

Now the above solutions match when

$$C_1 = 1, \quad C_2 = -\frac{16\pi G_{D+1}}{(D-1)\Omega_{D-1}} \equiv -\alpha_D M. \tag{2.16}$$

Thus, the metric of a spherical black hole in the asymptotically AdS_{D+1} space is

$$\begin{aligned} \mathrm{d}s^2 &= -\left(\frac{r^2}{\mathcal{L}^2} + 1 - \frac{\alpha_D M}{r^{D-2}}\right)\mathrm{d}t^2 + \left(\frac{r^2}{\mathcal{L}^2} + 1 - \frac{\alpha_D M}{r^{D-2}}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{D-1}, \\ \alpha_D &= \left(\frac{16\pi G_{D+1}}{(D-1)\Omega_{D-1}}\right), \end{aligned} \tag{2.17}$$

where the parameter M is related to the mass of the black hole. Note that the solution is characterized by two variables: the mass M and the AdS scale $\mathcal{L}$.

2.1.2.1 Hawking temperature

In 1974, Hawking realized that black holes are not completely black but can radiate [32, 33]. Further, the radiation was found to be perfectly black body radiation and, thus, black holes have well-defined temperature called a Hawking temperature. The existence of the Hawking temperature implies that black holes can carry entropy. This entropy is called a Bekenstein-Hawking entropy [34]. For a $D+1$ dimensional black hole the entropy is

$$S_{\mathrm{BH}} = \frac{A_{D-1}}{4G_{D+1}}, \tag{2.18}$$

where is A_{D-1} is the volume of the black hole horizon and G_{D+1} is Newton's constant in $D+1$ dimensions.

For the general type of metrics

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + g(r)^{-1}\mathrm{d}r^2 + \ldots, \tag{2.19}$$

where the functions $f(r)$ and $g(r)$ vanish at the horizon $r = r_0$[3], the Hawking temperature can be found by considering euclidean time ($\tau = -it$) and introducing a coordinate transformation, such that the metric near the horizon looks like a cylinder. To avoid the coordinate singularity at horizon, the euclidean time must have period 2π. This periodicity in the euclidean time can be identified with inverse temperature which fixes the Hawking temperature for the black hole [35].

A calculation of the Hawking temperature goes as follows. The near horizon expansion for the metric (2.19) is

$$\mathrm{d}s^2 \approx -f'(r_0)(r-r_0)\mathrm{d}t^2 + (g'(r_0)(r-r_0))^{-1}\mathrm{d}r^2 + \ldots. \tag{2.20}$$

[3]More precisely it is the largest root of the functions $f(r)$ and $g(r)$.

Next, consider a coordinate transformation $\mathrm{d}\rho = (g'(r_0)(r-r_0))^{-\frac{1}{2}}\mathrm{d}r$ for which the integrated form is

$$r = \frac{4r_0 + g'(r_0)\rho^2}{4}. \tag{2.21}$$

This transforms the above metric to

$$\mathrm{d}s \approx \frac{f'(r_0)g'(r_0)\rho^2}{4}\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \ldots, \tag{2.22}$$

for which the identification $\mathrm{d}\phi = \left[\frac{f'(r_0)g'(r_0)}{4}\right]^{\frac{1}{2}}\mathrm{d}\tau$ will take the metric to cylinder coordinates. Now, since the period of the euclidean time is related to inverse temperature [35], one obtains that

$$\begin{aligned} 2\pi &= \frac{1}{2}\left[f'(r_0)g'(r_0)\right]^{\frac{1}{2}} T_{BH}^{-1} \quad \Rightarrow \\ T_{BH} &= \frac{\left[f'(r_0)g'(r_0)\right]^{\frac{1}{2}}}{4\pi}. \end{aligned} \tag{2.23}$$

Using the above formula, the Hawking temperature for the AdS black hole metric (2.17) is

$$T_{BH} = \frac{1}{4\pi}\left[\frac{2r_0}{\mathcal{L}^2} + (D-2)\frac{\alpha_D M}{r_0^{D-1}}\right], \tag{2.24}$$

which can be written in a slightly different form by using the equation $f(r_0) = 0$:

$$T_{BH} = \frac{1}{4\pi}\frac{Dr_0^2 + (D-2)\mathcal{L}^2}{r_0\mathcal{L}^2}. \tag{2.25}$$

Note that there are two scales entering to the temperature formula which are the horizon position r_0 and the AdS scale $\mathcal{L}$.

2.1.2.2 Planar limit

The euclidean metric (2.17) for which the corresponding Hawking temperature is (2.25) has a topology $\mathrm{S}^1 \times \mathrm{S}^{D-1}$ at fixed r, by using a planar limit [36] this can be deformed to $\mathrm{S}^1 \times \mathrm{R}^{D-1}$. Thus, taking the planar limit corresponds to identification

$$\mathrm{d}\Omega_{D-1}^2 \sim \sum_{n=1}^{D-1}(\mathrm{d}x^i)^2. \tag{2.26}$$

This limit is directly related to D-brane solutions found in string theory that will play a crucial role in constructing the AdS/CFT duality.

The idea of the planar limit is to make the radius of S^{D-1} sphere (r_{D-1}) to be much larger than the radius of euclidean time r_τ. Then

$$\frac{r_\tau}{r_{D-1}} \to 0, \tag{2.27}$$

and the topology of the solution could be regarded as $\mathrm{S}^1 \times \mathrm{R}^{D-1}$.

To obtain the above behavior, introduce a scaling [30]:

$$r \rightarrow \lambda^{\frac{1}{D}} r, \tag{2.28}$$

$$t \rightarrow \lambda^{-\frac{1}{D}} t, \tag{2.29}$$

where

$$\lambda \equiv \left[\frac{\alpha_D M}{\mathcal{L}^{D-2}}\right]^{\frac{1}{D}}. \tag{2.30}$$

Then the rescaled metric is

$$\mathrm{d}s^2_{\mathrm{AdS}} = -\left(\frac{r^2}{\mathcal{L}^2} + \lambda^{-2} - \frac{\mathcal{L}^{D-2}}{r^{D-2}}\right)\mathrm{d}t^2 + \left(\frac{r^2}{\mathcal{L}^2} + \lambda^{-2} - \frac{\mathcal{L}^{D-2}}{r^{D-2}}\right)^{-1}\mathrm{d}r^2 + r^2\lambda^{2/D}\mathrm{d}\Omega^2_{D-1}. \tag{2.31}$$

Taking a limit $\lambda \rightarrow \infty$, which is obtained by taking $M \rightarrow \infty$, leads to the metric and the Hawking temperature that are

$$\mathrm{d}s^2_{\mathrm{AdS}} = -\left(\frac{r^2}{\mathcal{L}^2} - \frac{\mathcal{L}^{D-2}}{r^{D-2}}\right)\mathrm{d}t^2 + \left(\frac{r^2}{\mathcal{L}^2} - \frac{\mathcal{L}^{D-2}}{r^{D-2}}\right)^{-1}\mathrm{d}r^2 + r^2\lambda^{2/D}\mathrm{d}\Omega^2_{D-1}, \tag{2.32}$$

$$T_{BH} = \frac{Dr_0}{4\pi\mathcal{L}^2}, \quad r_0 = \mathcal{L}. \tag{2.33}$$

Now the ratio of the two radii near the boundary ($r \rightarrow \infty$) is

$$\frac{r_\tau}{r_{D-1}} \propto \frac{1}{\lambda^{1/D}} \rightarrow 0 \text{ when } \lambda \rightarrow \infty, \tag{2.34}$$

that is exactly what is needed, since now one can regard $r^2\lambda^{2/D}\mathrm{d}\Omega^2_{D-1}$ as a sphere with a very large radius, much larger than the AdS scale $\mathcal{L}$. This implies that one make a identification where

$$r^2\lambda^{2/D}\mathrm{d}\Omega^2_{D-1} \rightarrow \frac{r^2}{\mathcal{L}^2}\mathrm{d}x_i\mathrm{d}x^i. \tag{2.35}$$

Therefore, the topology of the black hole solution has "deformed"

$$\mathrm{S}^1 \times \mathrm{S}^{D-1} \;\Rightarrow\; \mathrm{S}^1 \times \mathrm{R}^{D-1}. \tag{2.36}$$

Finally, the planar limit for the AdS black hole solution is

$$\mathrm{d}s^2_{\mathrm{planar}} = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + \frac{r^2}{\mathcal{L}^2}\mathrm{d}x_i\mathrm{d}x^i, \tag{2.37}$$

$$f(r) \equiv \frac{r^2}{\mathcal{L}^2}\left(1 - \frac{\mathcal{L}^D}{r^D}\right),$$

which has the boundary at $r \rightarrow \infty$ and the horizon at $r = \mathcal{L}$. This solution corresponds to euclidean space with the Hawking temperature $T_{BH} = \frac{Dr_0}{4\pi\mathcal{L}^2} = \frac{D}{4\pi r_0}$. Notice, that in the planar limit, the metric is characterized only by the value $\mathcal{L}$ and the explicit dependence on the mass M has disappeared. Near the boundary ($r \rightarrow \infty$), the planar black hole metric can be directly related to the empty AdS space (2.3) by introducing a coordinate transformation $r \rightarrow \mathcal{L}^2/z$.

2.1.3 CFT

Conformal field theory (review [31]) is a field theory that is symmetric under conformal transformations. In particular, the conformal field theory has in addition to the Poincaré symmetry, a scaling symmetry and a special conformal symmetry so that the full symmetry group of the theory is $\mathrm{SO}(D,2)$. The additional scaling symmetry implies that the theory must be free of any dimensionful parameters such as gauge coupling g or mass m. Note that a theory may be classically invariant under the conformal transformations but quantum effects and self-interactions might break the symmetry by dynamically generating a mass scale to the quantum theory. For example, quarkless QCD has a classical conformal symmetry but, due to quantum effects, it dynamically generates a mass scale $\Lambda_{\mathrm{QCD}} \approx 200$ MeV so that the conformal symmetry is absent in quantum theory.

Two classic examples of CFTs are the two dimensional string worldsheet theory and the $\mathcal{N}=4$ superconformal Yang-Mills theory in four dimensions. In general, CFTs have great simplifications compared to ordinary Yang-Mills theories. For example, in the worldsheet theory, correlators such as $\langle\mathcal{O}(x)\mathcal{O}(y)\rangle$ can be obtained by just studying the scaling dimensions of the operators

$$\langle\mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{C}{(x-y)^{2h}}, \tag{2.38}$$

where h is the scaling dimension of the primary field (operator) $\mathcal{O}(x)$, which depends on its mass dimension and spin. The coefficient C is specific for a theory.

Note that global symmetry of the $D+1$ dimensional AdS space ($\mathrm{SO}(D,2)$) is the same as the conformal group in D dimensional flat space.

2.1.4 SUSY

A theory is supersymmetric if it is invariant under supersymmetry transformations [37]. The supersymmetry (SUSY) transformations act on the fields and change bosons to fermions and vice versa:

$$\text{bosons} \ \Leftarrow \text{SUSY} \Rightarrow \ \text{fermions}$$

In addition to the Poincaré generators, supersymmetric field theory has supersymmetry generators that are spinors (half integer), called supercharges. A supersymmetric theory containing $\mathcal{N}$ supercharges has a global $\mathrm{U}(\mathcal{N})_{\mathrm{R}}$ symmetry. This symmetry acts on the supersymmetry generators and is called a R-symmetry. Adding supersymmetry to a field theory can simplify the theory, since it may prevent quantities like the gauge coupling from getting any quantum corrections. Supersymmetry is one of the scenarios introduced to explain some of the physics beyond the standard model and, in particular, it can be used to cure the hierarchy problem related to the Higgs mechanism. The study of low energy supersymmetry is one of the main goals of the LHC.

One can also take one step further and gauge the supersymmetry. Gauging means that one takes the symmetry transformation to be a function of the spacetime coordinates, so that the transformation is different on each spacetime point. For example, gauging the $\mathrm{U}(1)$ symmetry of the Dirac Lagrangian

$$S_{\mathrm{Dirac}} = \int \mathrm{d}^{D+1}x\, \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi, \tag{2.39}$$

leads to QED, where the Dirac spinor (electron) has interactions with the gauge field (photon). Similarly, gauging supersymmetry leads to supergravity (SUGRA) [38]. Supergravity contains spin-two particles that are identified with the gravitons and the low energy theory of the supergravity should lead to (linearized) Einstein equations.

When one considers a theory that possesses both supersymmetry and conformal symmetry, then the theory is said to be superconformal. A superconformal theory has ordinary conformal and supersymmetric generators, but also new superconformal generators that are needed to close the generator algebra.

2.1.5 Superconformal field theory, an example

A superconformal theory that has an important role in the AdS/CFT conjecture, is the four dimensional $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory [26]. This theory possesses conformal invariance even at the quantum level. The action for the theory can be found by supersymmetrizing the SU(N) Yang-Mills theory, and its bosonic part takes a form

$$S_{\mathcal{N}=4,\,\mathrm{SYM}} = \frac{1}{g_{\mathrm{YM}}^2} \int \mathrm{d}^4x \,\mathrm{Tr}\left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2} D_\mu \Phi^I D^\mu \Phi^I - \frac{1}{4} \left[\Phi^I, \Phi^J\right]^2 \right] + \dots \tag{2.40}$$

This action has the Poincaré and the scaling symmetry implying symmetry under the full conformal group, which in four dimensions is SO(4,2). However, the theory contains fermions and hence the bosonic conformal group SO(4,2) must be enlarged to SU(2,2). In addition, there is a global R-symmetry that acts to the four supercharges. This symmetry group is $\mathrm{SU}(4)_\mathrm{R}$ for which the bosonic counterpart is $\mathrm{SO}(6)_\mathrm{R}$. The product of two symmetry groups SU(2,2) and $\mathrm{SU}(4)_\mathrm{R}$ is the superconformal symmetry group denoted by SU(2,2|4). Finally, the symmetry group of the four dimensional $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory is given by [27]

$$\mathrm{SO}(4,2) \times \mathrm{SO}(6)_\mathrm{R} \sim \mathrm{SU}(2,2) \times \mathrm{SU}(4)_\mathrm{R} = \mathrm{SU}(2,2|4), \tag{2.41}$$

where the left hand side is the bosonic part and the right hand side is the fermionic counterpart. Later, it is shown that the boundary of $\mathrm{AdS}_5 \times \mathrm{S}^5$ has exactly the same symmetry group as this theory. This will be the first consistency check of the AdS/CFT conjecture.

2.1.6 String theory and D-brane solutions

In (super)string theory [39, 40] elementary particles are identified with one dimensional strings (with a length l_s) living in a spacetime with ten or eleven dimensions. String theory was built to describe the strong interactions between the quarks. However, this string model of hadrons had some fatal problems. For example, it predicted spin 2 particles which were not seen in experiments. However, this failure was not the end of string theory, but it was reconsidered as a unifying theory for quantum field theories and gravity [41]. String theory was found to cure some problems that appear in ordinary QFTs. In particular, divergences at short distance are solved by considering particles as one-dimensional strings instead of just dimensionless points as is done in QFT. In addition, string theory provides a way to quantize the Einstein's theory of gravity where the quantum that mediates the gravitation interaction (the graviton) is a closed string with spin 2.

There are two types of strings in superstring theory: closed strings and open strings. The endpoints of the closed strings are attached to each other while the endpoints of the open strings are free to move in spacetime. As mentioned, one example of a closed string is the graviton and open strings can be considered as counterparts for the gauge bosons of ordinary QFT. Therefore, considering string theory in the limit where energies and the string length are small, the dynamics of the closed strings should be described by supergravity and open strings should be described by a (supersymmetric) gauge theory.

$$\begin{array}{ccc} & \text{limit: } E,\ \sqrt{\alpha'} = l_s \to 0 & \\ \text{Closed strings} & \Rightarrow & \text{Supergravity} \\ \text{Open strings} & \Rightarrow & \text{Supersymmetric gauge theory} \end{array}$$

In addition to the closed and the open strings, string theory contains extended objects called D-branes which were introduced by Polchinski [42]. There are two separate ways to study D-branes in string theory. One way, is to consider them as manifolds embedded in some higher dimensional space and, since they are massive, the D-brane curves the spacetime around them.

Another way is to consider D-branes as surfaces where the open string endpoints are attached. The name "D-brane" comes from the Dirichlet boundary conditions that open strings must satisfy on the surface of the brane. Hence, the endpoints of the open strings are not free to move in ten dimensional space but are restricted to move on the lower dimensional surface of the D-brane. More precisely, the Dp-branes have $p+1$ dimensions, thus, the number of fixed Dirichlet boundary conditions for open strings is $p+1$. Clearly, closed strings cannot have similar fixed boundary conditions on the D-brane and therefore are able to "leak" out from the sourcing brane and, thus, can curve the spacetime geometry next to the D-branes.

Consider the closed string interpretation in more detail. Since the low energy and zero string length approximation of the classical closed string theory is thought to be classical supergravity, there should exist a solution to supergravity equations which describes massive objects which can be identified with the D-branes. Actually, this is the case and the solution is found by considering ten dimensional IIB supergravity action which in the Einstein frame is [40]

$$S_{\text{SUGRA}} = \frac{1}{16\pi G_{10}} \int \mathrm{d}^{10}x\sqrt{-g}(R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\frac{1}{5!}F_5^2 + \ldots). \tag{2.42}$$

Here R is Ricci scalar, $G_{10} = 8\pi^6(\alpha')^4 g_s^2$, F_5 is the field strength for the four form C_4, and the dots denote fermionic terms, in addition to other Ramond-Ramond p-forms that are irrelevant for the particular solution. A D3-brane solution is found by considering a metric ansatz

$$\mathrm{d}s^2_{\text{Brane}} \quad = \quad -B(r)^2\mathrm{d}t^2 + E(r)^2\sum_{i=1}^{3}(\mathrm{d}x^i)^2 + R(r)^2\mathrm{d}r^2 + G(r)^2r^2\mathrm{d}\Omega_5^2, \tag{2.43}$$

where the coordinates (t, x^i) describe points on the D-brane and the coordinate r is the transverse distance from the D3-brane. The $\mathrm{d}\Omega_5^2$ part describes the S^5 unit sphere in the transverse space. The ansatz is clearly static and has a translation invariance in the coordinates (t, x^i). The components of the field strength $F_5 = \mathrm{d}C_4$ and dilaton ϕ are assumed to depend only on transverse distance r. Further, for type IIB supergravity one has to impose a self-duality condition $F_5 = \star F_5$.

The D3-brane solution is [43]

$$\begin{aligned} \mathrm{d}s^2 &= H(r)^{-\frac{1}{2}}\left(-g(r)\mathrm{d}t^2 + \sum_{i=1}^{3}(\mathrm{d}x^i)^2\right) + H(r)^{\frac{1}{2}}\left(g(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_5^2\right), \quad (2.44) \\ g(r) &= 1 - \left(\frac{r_0}{r}\right)^4, \; H(r) = 1 + \left(\frac{\mathcal{L}}{r}\right)^4, \; e^{\phi} = 1, \; \int_{\mathrm{S}^5}\mathrm{d}C_4 = N, \quad (2.45) \end{aligned}$$

which has a horizon at r_0. Interestingly, for N coincident branes, the parameter $\mathcal{L}$ can be related to the string length l_s and to the string coupling g_s with a relation [44]

$$\mathcal{L}^4 = 4\pi g_s l_s^4 N. \quad (2.46)$$

In the D3-brane solution the function $H(r)$ or in particular the parameter $\mathcal{L}$, divides the spacetime into two separate regions. For $r \gg \mathcal{L} > r_0$ the metric is asymptotically flat

$$\mathrm{d}s^2 = \left(-\mathrm{d}t^2 + \sum_{i=1}^{3}(\mathrm{d}x^i)^2\right) + \left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_5^2\right), \quad (2.47)$$

and for $\mathcal{L} \gg r > r_0$ the metric is

$$\mathrm{d}s^2 = \left(\frac{r}{\mathcal{L}}\right)^2\left(-g(r)\mathrm{d}t^2 + \sum_{i=1}^{3}(\mathrm{d}x^i)^2\right) + \left(\frac{\mathcal{L}}{r}\right)^2 g(r)^{-1}\mathrm{d}r^2 + \mathcal{L}^2\mathrm{d}\Omega_5^2. \quad (2.48)$$

This can be also written as

$$\begin{aligned} \mathrm{d}s^2 &= -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + \frac{r^2}{\mathcal{L}^2}\mathrm{d}x_i\mathrm{d}x^i + \mathcal{L}^2\mathrm{d}\Omega_5^2, \quad (2.49) \\ f(r) &\equiv \frac{r^2}{\mathcal{L}^2}g(r) = \frac{r^2}{\mathcal{L}^2}\left(1 - \left(\frac{r_0}{r}\right)^4\right). \end{aligned}$$

The first part of the above metric is the same as the planar limit of the asymptotically AdS_5 black hole (2.37) with the AdS scale $\mathcal{L}$. The second part ($\mathcal{L}^2\mathrm{d}\Omega_5^2$) is the metric for the five sphere S^5 with a radius $\mathcal{L}$. Thus, the limit $\mathcal{L} \gg r > r_0$ of the D3-brane metric is

$$\mathrm{AdS}_5(\mathcal{L}) \times \mathrm{S}^5(\mathcal{L}). \quad (2.50)$$

The D3-brane was a solution to classical IIB supergravity, which is the approximation of IIB string theory. Next one should check that the approximation is valid for the particular solution. To suppress stringy effects, one must demand the AdS scale to be much greater than the string scale. In addition, quantum effects can be considered to be negligible if the dimensionless combination of Newton's constant and AdS scale

$$G_{10}/\mathcal{L}^8 \; \propto \; g_s^2(\alpha')^4/\mathcal{L}^8 = g_s^2 l_s^8/\mathcal{L}^8 \quad (2.51)$$

is small enough. These together imply that the D3-brane solution must have

$$\mathcal{L} \gg l_s, \ \frac{g_s^2(\alpha')^4}{\mathcal{L}^8} \ll 1. \tag{2.52}$$

Further, using the relation (2.46), these conditions can be written as

$$4\pi g_s N \gg 1, \ \frac{1}{N} \ll 1, \tag{2.53}$$

which are satisfied (at least) for $N \to \infty$ and $g_s \to \mathcal{O}(1)$.

In the above, the D3-brane solution was introduced as an instanton type of solution to the supergravity equations. Next consider the open string interpretation where the D3-branes are identified as hypersurfaces where endpoints of open strings are stuck. In the perturbative picture ($g_s \ll 1$), the dynamics of single D3-brane are covered by a Dirac-Born-Infeld (DBI) action whose bosonic part is

$$S_{D3} = -\frac{1}{g_s(2\pi)^3 l_s^4} \int \mathrm{d}^4\sigma\, e^{-\phi} \sqrt{-\det\left[g_{ab} + B_{ab} + 2\pi\alpha'^2 F_{ab}\right]} + S_{cs} \tag{2.54}$$

where the fields are pullbacks of the corresponding ten dimensional spacetime fields to D-brane worldvolume [28, 45]. By choosing a particular gauge and assuming B_{ab}, ϕ to vanish and, finally, expanding in α', this can be written as

$$S = -\frac{1}{g_s(2\pi)^3(\alpha')^2} \int \mathrm{d}^4x \left(\frac{1}{2}\partial_\mu X^M \partial^\mu X_M + \frac{(2\pi\alpha')^2}{4} F_{\mu\nu}F^{\mu\nu}\right) + \ldots. \tag{2.55}$$

This looks like a four dimensional U(1) (supersymmetric) gauge theory and in fact, it is the bosonic part of the $\mathcal{N} = 4$ supersymmetric U(1) Yang-Mills theory with a gauge coupling

$$g_{YM}^2 \equiv 2\pi g_s. \tag{2.56}$$

The above discussion was made for a single D3-brane but it can be generalized to N coincident D3-branes for which the dynamics are covered by the action

$$S_{\text{bos}} = \frac{1}{g_{YM}^2} \int \mathrm{d}^4x \, \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\Phi^I D^\mu\Phi^I - \frac{1}{4}\left[\Phi^I, \Phi^J\right]^2\right] + \ldots, \tag{2.57}$$

where the dots include the fermionic degrees of freedom. This action is familiar from the subsection 2.1.5 and it is the $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory with a gauge coupling

$$g_{YM}^2 \equiv 2\pi g_s. \tag{2.58}$$

The regime where the perturbative open string interpretation is valid is simply $g_{YM} \ll 1$. Statements about the validity of classical supergravity approximation (2.53) can now be reconsidered by using the gauge theory variables (2.58). This leads to equations

$$g_{YM}^2 N \gg 1, \ \frac{1}{N} \ll 1, \tag{2.59}$$

where the first equation is so called large 't Hooft limit of the gauge theory, where the 't Hooft coupling is defined as $\lambda = g_{YM}^2 N$. The second condition implies that the number of degrees of freedom in the gauge theory must be large. Since, the open string interpretation is valid only when the coupling is small ($\lambda \to 0$) and the supergravity approximation is valid only when the gauge coupling is strong ($\lambda \to \infty$) it seems that two interpretations are completely disconnected but surprisingly, as first recognized by Maldacena, this is not the case.

2.2 AdS/CFT duality

At first look, two interpretations of the D3-branes did not have much to do with each other but surprisingly, some calculations done in different interpretations seemed to match [29, 44, 46, 47]. Hence, there where hints that these two ways of studying D3-branes are related more closely. In fact, in 1997 Juan Maldacena [3] conjectured that the interpretations are equivalent. More precisely, the conjectured equivalence was between type IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and the four dimensional superconformal $\mathcal{N} = 4$ SU(N) Yang-Mills theory. Maldacena conjectured that instead of being two different theories they are in fact dual formulations of the same theory. In general, the Anti de Sitter factor forces the dual theory to be a conformal field theory, and the form of the compact manifold tells how supersymmetry is realized in the theory. Further, Maldacena conjectured dualities between string theory/M-theory on a class of backgrounds $\mathrm{AdS}_d \times \mathrm{X}^{D-d}$ and conformal field theories at the boundary of the Anti de Sitter space, where for string theory $D = 10$ and for M-theory $D = 11$ [3]. For D3-branes, the more precise conjectured equivalence between two theories and their parameters is:

- Ten dimensional type IIB superstring theory on $\mathrm{AdS}_5(\mathcal{L}) \times \mathrm{S}^5(\mathcal{L})$ with Newton's constant $G_{10} = 8\pi^6(\alpha')^4 g_s^2$ and string length l_s
- Four dimensional $\mathcal{N} = 4$ super Yang-Mills theory with gauge group SU(N) and coupling g_{YM}
- The relations between the parameters are $g_{YM}^2 = 2\pi g_s$ and $\mathcal{L}^4 = 4\pi g_s l_s^4 N$

The conjectured duality acts between full quantum theories and can be expressed by a fundamental relation [7, 8]:

$$Z_{\mathcal{N}=4,\,\mathrm{SYM}}\left[J_i(x)\right] = Z_{\mathrm{IIB},\,\mathrm{AdS}_5\times\mathrm{S}^5}\left[J_i(x,z)\right]_{J_i(x,z=0)=J_i(x)}\,. \tag{2.60}$$

This relates the theories by their euclidean path integrals. The left hand side ($Z_{\mathcal{N}=4,\,\mathrm{SYM}}$) is the generating functional of the CFT where the fields $J_i(x)$ act as sources for the gauge invariant operators $\mathcal{O}(x)$. The right hand side is the partition function for IIB string theory in the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background. On this side, the fields $J_i(x,z)$ have fixed value $J_i(x)$ at $z \to 0$, where $z = 0$ represents the conformal boundary of the AdS_5 space.

At this point one can make simple checks about the validity of the conjecture. One possible way is to compare the symmetries of theories. In particular, the (bosonic) symmetry group of the string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ background is simply $\mathrm{SO}(4,2) \times \mathrm{SO}(6)$. If the conjectured equivalence between the theories is true, the symmetry group of the conformal YM theory should be the same. Indeed, as it was found in subsection 2.1.5, for the four dimensional superconformal $\mathcal{N} = 4$ SU(N) the symmetry group is the same $\mathrm{SO}(4,2) \times \mathrm{SO}(6)$. Although the conjectured equivalence between the theories has passed all the tests done ever since it was introduced, a rigorous mathematical proof of the conjecture is still lacking [48].

2.2.0.1 't Hooft limit

The equivalence between string theory and conformal SYM theory was conjectured between complete quantum theories but there is also a non-trivial limit of the duality where many calculations can be done. This is the 't Hooft limit [11]. At the end of the subsection 2.1.6 it was found out, that the large 't Hooft limit may create a link between classical supergravity and strongly coupled gauge theory. Now let us consider this connection in more detail.

In the SU(N) gauge theory the 't Hooft limit corresponds to the topological expansion in $1/N$ where the 't Hooft coupling $\lambda = g_{YM}^2 N$ is kept fixed while the number of degrees of freedom is taken to infinity ($N \to \infty$). When considering the 't Hooft limit in the AdS side of the duality, one obtains that the string coupling can be written as $g_s \propto \lambda/N$. Hence in the limit $N \to \infty$, the string coupling vanishes ($g_s \to 0$) which corresponds to the classical approximation of string theory.

One can still go one step further and take $\lambda \to \infty$. This limit is the large 't Hooft coupling limit and it corresponds to the strongly coupled gauge theory. In string theory, this limit takes the AdS scale $\mathcal{L}$ to be much greater than string length l_s. This implies that the interesting curvature scales are much greater than the string scale. Thus, together with $g_s \to 0$, one has $l_s \to 0$ which reduces string theory to classical supergravity. To summarize:

- 't Hooft limit: λ fixed and $N \to \infty$ corresponds to classical string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and to the $\frac{1}{N}$ -expansion of the gauge theory
- Large 't Hooft coupling limit: $\lambda \to \infty$ corresponds to classical supergravity on $\mathrm{AdS}_5 \times \mathrm{S}^5$ and to the strongly coupled gauge theory.

In the large 't Hooft coupling limit, the partition function for string theory can be approximated by a equation [36]

$$Z_{\mathrm{IIB,\,AdS_5\,\times\,S^5}} \approx e^{-S_{\mathrm{SUGRA}}}, \tag{2.61}$$

where S_{SUGRA} is the classical action of the IIB supergravity. Hence, the fundamental relation (2.60) in this limit is given by a relation

$$Z_{\mathcal{N}=4,\,\mathrm{SYM},\,\lambda\to\infty,\,N\to\infty} = e^{-S_{\mathrm{SUGRA}}}. \tag{2.62}$$

Now one can relate the scaling dimensions of the SYM operators ($\triangle$) to masses of the supergravity (scalar-)fields on AdS_{D+1} by a relation [7]

$$\triangle = \frac{D}{2} + \sqrt{\frac{D^2}{4} + m^2}. \tag{2.63}$$

The AdS/CFT duality comes with a "dictionary" that tells how to map classical supergravity fields to the corresponding gauge invariant operators and what are the exact relations between them [3, 7, 8]. For example, the holographic dictionary tells that the massless scalar on AdS_{4+1} is dual to the operator with a scaling dimension $\triangle = 4$ that can be identified with the gauge theory operator $\mathrm{Tr}\,[F^{\mu\nu}F_{\mu\nu} + \ldots]$. Another important relation is the mapping between the boundary metric $g_{\mu\nu}$ and the stress tensor $T_{\mu\nu}$ of the corresponding CFT.

In the next subsection a holographic method for calculating thermodynamics of the hot strongly coupled gauge theory is shown.

2.2.0.2 Thermodynamics

One way to test this duality is to try to use it to calculate the thermodynamical properties of the strongly coupled $\mathcal{N}=4$ SU(N) Yang-Mills theory. The holographic dictionary tells that the thermal field theory corresponds to the AdS space with a black hole [36]. Further, the thermodynamics of black hole are related to field theory thermodynamics. One simple way[4] to calculate the entropy of strongly coupled SYM is to use the Bekenstein-Hawking formula (2.18):

$$S_{\mathrm{BH}} = \frac{A}{4G}. \tag{2.64}$$

For D3-brane solutions [44] this is

$$S_{\mathrm{BH}} = \frac{A}{4G_{10}}, \tag{2.65}$$

where

$$A = \int \mathrm{d}^3x \int \mathrm{d}\Omega_5 \sqrt{-\gamma}|_{r=r_0} = \frac{\pi^2\mathcal{L}^8}{r_0^3}V_3. \tag{2.66}$$

Here V_3 is the volume of horizon. Substituting Newton's constant ($G_{10} = 8\pi^6(\alpha')^4 g_s^2$) and the Hawking temperature $T_{\mathrm{BH}} = \frac{1}{\pi r_0}$ to the above formula gives

$$S_{\mathrm{BH}} = \frac{\pi^2}{2^5\pi^6}\left(\frac{\mathcal{L}^4}{(\alpha')^2 g_s}\right)^2 (\pi T_{\mathrm{BH}})^3\, V_3. \tag{2.67}$$

Now the implication of the duality is that one is able to rewrite this formula using the dual field theory variables. The holographic dictionary sets $T_{\mathrm{BH}} = T_{\mathrm{YM}} \equiv T$ and the entropy of strongly coupled SYM is

$$S_{\mathrm{YM,\,strong}} = \frac{\pi^2}{2}N^2V_3T^3. \tag{2.68}$$

In the other hand, the entropy of SYM can be found by counting the degrees of freedom of asymptotically free theory. In particular, for $\mathcal{N}=4$ SYM counting leads to entropy [29]

$$S_{\mathrm{YM,\,weak}} = \frac{4}{3}\frac{\pi^2}{2}N^2V_3T^3, \quad S_{\mathrm{YM,\,weak}} = \frac{4}{3}S_{\mathrm{YM,\,strong}}. \tag{2.69}$$

There is something very interesting in this result. The entropy of the field theory is calculated using two totally different setups and the results are not equal but almost and, indeed, this small difference between entropies can be understood. The holographic calculation was done in the classical gravity approximation which is valid only for strong coupling ($\lambda \gg 1$) and, in the equation above, the is compared with entropy calculated in the limit of zero coupling ($\lambda \to 0$). Therefore, the difference between the entropies is not a surprise [5].

2.2.1 Generalizations of the AdS/CFT

Maldacena's $\mathrm{AdS}_5/\mathrm{CFT}_4$ conjecture had a large impact on theoretical physics. It was realized that it or more generally its modifications, could be used to model various phenomena and,

[4] To do this properly one should calculate the on-shell action and regulate it. See [49, 50, 51].

[5] Corrections to entropy formulas can be found in [29].

hence, it brought physicists from very different areas to study its possibilities. A "shortcoming" of the original conjecture is that the dual field theory has a superconformal symmetry, which is something not seen in nature. Thus, it became clear that the original duality must be modified to be able to study more realistic field theories. For example, so far dualities for theories with less supersymmetry, broken conformal symmetry, broken chiral symmetry, spontaneous symmetry breaking, etc, have been studied. The different modifications and extensions of the original duality generally go under the name of gauge/gravity duality.

The goal of this section is to study the large-N QCD-type theories by using the gauge/gravity duality and, hence, only some basic properties of QCD-type theories and their holographic counterparts are shortly introduced. Using gauge/gravity duality in slightly different setups can be found in [53, 54, 55, 56, 57, 58] and references therein.

There are generally three ways to get something similar to QCD out from the gauge/gravity duality. One way is to introduce small modifications to the original conjecture. One of such modification was studied above and that was the addition of black hole to AdS space which corresponded to thermal gauge theory [36]. Further, this temperature breaks supersymmetry and so it is more close to QCD. As discussed in the introductory section, this can be used to study quark-gluon plasma produced at RHIC and LHC, where the properties[6] of plasma are approximated by the strongly coupled limit of thermal QCD, see [59] and references therein. Still, there are major differences between thermal QCD and thermal $\mathcal{N}=4$ SYM. One such is the lack of particles which corresponds to the quarks in QCD. The QCD quark is a particle transforming in the fundamental representation under the gauge group SU(N), but the fermions of supersymmetric theory are just superpartners of gauge fields and, hence, transform as an adjoint field.

One way[7] to introduce fundamental quarks to AdS/CFT is to add D7-branes to the AdS setup. The perturbative picture of the D3-D7 system includes, in addition to open strings that have both endpoints on the D3-branes, also open strings that have one endpoint on the D3-brane and another on D7-brane. Further, the endpoint of the open string which ends on the stack of D3-branes, transforms under the fundamental representation of the gauge group SU(N) and, hence, it can be identified with the QCD quark. The field theory dual this system is $\mathcal{N}=2$ SU(N) SYM [19]. The action for the D3-D7 system is

$$S = S_{\mathrm{IIB,\,AdS_5\times S^5}} + S_{\mathrm{D7}} \tag{2.70}$$

where the dynamics of the D7 brane(s)[8] is covered by DBI action

$$S_{\mathrm{D7}} = -\frac{1}{(2\pi)^7 g_s(\alpha')^4}\int \mathrm{d}^8\zeta\,\sqrt{-\det\left[G_{ab}+2\pi\alpha' F_{ab}\right]} + S_{cs} \tag{2.71}$$

where S_{cs} is a Chern-Simons action [45]. Unfortunately, solutions to the D3-D7 system are found only when D7-branes are introduced as a small perturbation over the background generated by the D3-branes. Thus, D7-branes do not change the AdS_5 background geometry. This perturbative limit to D3-D7 limit is called a probe limit [60] and it implies that the number of D3 branes is assumed to be much greater than D7 branes ($N_3 \gg N_7$)[9]. Thus the

[6]Nearly conformal and strongly coupled. The holographic calculations are still done in the large N whereas QCD have only $N=3$.

[7]See [19, 20, 21, 60, 61, 62].

[8]For multiple D7-branes one should consider the non-Abelian versions of DBI action.

[9]The number of gauge fields is much greater than the number of fundamental fermions.

probe limit is in contradiction to QCD where these number are roughly the same. Altogether, the above construction is quite far from QCD since there is nothing like the QCD scale ($\Lambda_{\rm QCD}$) in this setup.

Another way to get something similar to QCD is to start with a completely different setup that does not include D3-branes at all but, instead use D4-branes as was done in [20, 21]. The D4-branes are classical solutions to type IIA string theory. For a D4-brane system the background geometry is

$$\mathrm{d}s^2 = \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + f(U)\mathrm{d}\tau^2\right) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{\mathrm{d}U^2}{f(U)} + U^2\mathrm{d}\Omega_4^2\right), \tag{2.72}$$

$$e^\phi = g_s\left(\frac{U}{R}\right)^{3/4}, \quad F_4 = \mathrm{d}C_3 = \frac{2\pi N_c}{V_4}\epsilon_4, \quad f(U) = 1 - \frac{U_{\rm KK}^3}{U^3}. \tag{2.73}$$

Note that the geometry is not asymptotically AdS and the dual supersymmetric field theory is five dimensional ($x^\mu = x^0 ... x^3$ and τ). To get something like four dimensional QCD, one must compactify one of five dimensions to circle (τ), so that the boundary conditions in the compactified dimension breaks supersymmetry. Further, the metric has a scale ($U_{\rm KK}$) which is analog to the QCD scale $\Lambda_{\rm QCD}$. Hence, at low energies, the dual gauge theory is similar to the quarkless QCD and for example the masses of glueballs can be calculated, which are found to roughly agree with those found in lattice QCD simulations, see [19] and references therein.

Fundamental matter can be added to the system similarly to the D3-D7 setup. In this case, one adds a small number of D8-branes and anti D$\bar{8}$-branes to the background generated by the D4-branes [20, 21]. In this setup one is able to study chiral symmetry breaking by using classical gravity. Although this is closer to QCD than the D3-D7 system, there are still many differences. In particular, one common problem in these type of "top-down" setups is that the scale of the low energy excitations (glueballs, mesons and Kaluza-Klein masses) is close to the length scale of the compactified dimensions and so the dual field theory cannot really be considered as four dimensional.

There are also more phenomenological constructions. These are known as AdS/QCD or bottom-up approaches. The logic in these models is inverse to top-down models, where the gravity part was some string theory on a curved manifold which had a specific field theory dual. In the AdS/QCD models, one starts with the known properties of QCD-type theories and tries to figure out which are the corresponding fields in the gravity background. There are two well know classes of AdS/QCD. One class is called hard-wall models [63] where the gravity background has a "wall" where the space ends. This introduces a QCD-type scale to the corresponding field theory. One of the simplest hard wall models has a following five dimensional action:

$$S = \int \mathrm{d}^5x\,\sqrt{-g}\mathrm{Tr}\left[-|D\Phi| + 3|\Phi| - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right] \tag{2.74}$$

where the gauge fields A_L^μ and A_R^μ are dual to the QCD operators $\bar{q}_L\gamma^\mu q_L$ and $\bar{q}_R\gamma^\mu q_R$ whereas the scalar Φ is dual to the operator $\bar{q}_R q_L$. The five dimensional background is

$$\mathrm{d}s^2 = \frac{\mathcal{L}^2}{z^2}\left(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}z^2\right) \tag{2.75}$$

where the range of the AdS coordinate is taken to be finite, i.e., $0 < z < z_m$. The energy scale of the dual field theory is now set by a condition $z_m \sim \Lambda_{\rm QCD}$. This and its successors encodes some properties of QCD surprisingly well [19].

In general, the problem with hard wall models is that the meson mass spectrum grows with respect to excitation number a

$$M_n^2 \sim n^2, \tag{2.76}$$

which contradicts QCD where the dependence is linear ($M_n^2 \sim n$). This problem can be rather easily cured by "smoothening" the wall. This can be done by taking the conformal factor $\frac{\mathcal{L}^2}{z^2}$ to be some smooth function

$$e^{2\phi(z)} \tag{2.77}$$

which, with a certain profile, can reproduce the linear behavior for the meson mass spectrum. These type of constructions are generally called a soft-wall models [64].

The major problem with a soft-wall models is that the gravity background is not a solution to any Einstein equations. Thus, the background is not dynamical. One implication of this fact, is that there is no clear way how to calculate the thermodynamics of corresponding field theory. In the hard-wall models there is another contradiction with QCD, which is that in these models the magnetic charges are screened instead of anti screened [23].

There is a way to solve many problems of with bottom-up approaches by making the background dynamical. In the next section, we study a specific model called the 5D Improved Holographic QCD (IHQCD) [22, 23, 24] which lies somewhere between the top-down and bottom-up approaches.

Chapter 3

5D Einstein-Dilaton model

In the seventies, 't Hooft realized that gauge theories may be simpler within the limit where the number of colors N is large. In particular, this provides a new method of doing calculations, where the expansion parameter is not the naive Yang-Mills coupling constant g_{YM} but instead an expansion is performed in $1/N$. The large-N limit [11] can be used to extract information about gauge theories in the regime where the original perturbation theory fails. After Maldacena's conjecture [3], the power of the large-N expansion has become even more clear. The original conjecture has been generalized to gauge theories that are similar to theories seen in nature. After over ten years of study, holographic counterparts for field theory phenomena, where the strong IR dynamics are important, are quite well understood. For example, dual models have been constructed for confinement, chiral symmetry breaking.

In this thesis, a specific holographic model used to study large-N gauge field theories, is the 5D Improved Holographic QCD (IHQCD) model introduced by Kiritsis, Nitti and Gursoy [22, 23, 24]. The starting point of the model is 5D non-critical string theory [65]. In non-critical string theories the background curvatures and other geometric invariants are generally of the same order as the string scale. This implies that the two derivative effective action for gravity is not a valid approximation and one should instead use a higher derivative action[1]. Furthermore, an expansion in α' is not a reliable approximation anymore. Although, there are several unsolved issues in non-critical string theories there exist several attempts to construct holographic models based on such theories. Further, some results indicates that, although stringy effects can be important, some of the qualitative features of the system can be reproduced in field theory approximations. IHQCD is also based on this assumption [65].

The action of IHQCD contains only fields with two derivatives and the α' corrections leads to a non-trivial dilaton potential. Of course the dilaton potential cannot be calculated to arbitrary order in α'. Thus, in the phenomenological spirit of AdS/QCD, it is fixed by requiring that it reproduces the holographic counterparts to some known phenomena present in the large-N gauge field theories. This construction sets IHQCD to lie somewhere between the bottom-up and the top-down approaches discussed in subsection 2.2.1.

The above problems lead to the conclusion that IHQCD can be used to study the large-N gauge theories only at the qualitative level. In particular, the model can explain some of the qualitative features of the field theories but sharp predictions cannot be made. In spite of the above reservations, the IHQCD model studied in [66] was able to give remarkably good

[1] Higher derivatives come from stringy corrections.

fits for pure YM thermodynamics and also to glueball spectra compared to results found in other holographic setups. The IHQCD model gives some understanding of the transport properties of strongly coupled YM plasma where no reliable calculations have been made [67]. One example of the quantitative predictions of the model, is the observation that the IR θ-angle in large-N YM theory vanishes [23].

To construct holographic duals for more realistic field theories, for example QCD, one must add fundamental matter (quarks) to the string setup. Like in the examples of subsection 2.2.1, in IHQCD it can be done by adding D4-D$\bar{4}$ branes to the system [23, 68, 69]. Using this construction, one is able to study meson spectra and chiral symmetry breaking. Until recently [70], it has been possible only in the limit where the number of flavor fields is much smaller than N, i.e, in the probe limit.

In this thesis, the IHQCD model is extended to model, not only quarkless QCD (or pure YM), but various field theories where the effects of the fermions are generally known to be important[2]. Here, this is not achieved by adding the D4-D$\bar{4}$ branes to the system, but the effect of the fermions is assumed to lead to dilaton potentials that are different from the one used to model the pure YM [73, 74, 75] (articles II, III, IV). The validity of this method has not been carefully studied and, thus, the only criterion for its validity is the outcome of the analysis which must be compared to the known phenomena and results of the corresponding field theories.

In the section 3.1, IHQCD is introduced and some of its general properties are studied. In particular, a criterion for confinement is worked out and a holographic calculation of the glueball masses is presented. In the second section we concentrate to a specific IHQCD construction that is related to quarkless QCD and review some properties found in [66, 76]. The discussion of sections follows closely that in [22, 23, 24, 66, 76]. A simplified version of this specific IHQCD model was considered in the article I, in which we were able to perform analytic calculations which lead to similar results found by Kiritsis et al. In the last section 3.3 we study a generalization of IHQCD to quasi-conformal theories and give a short introduction to calculations performed in the articles II, III and IV including a study of the thermodynamics and the mass spectra of quasi-conformal theories.

3.1 Improved Holographic QCD (IHQCD)

The lowest-dimensional gauge invariant operators of quarkless QCD are the stress energy tensor $T_{\mu\nu} = \mathrm{Tr}\left[F_{\mu\alpha}F^{\alpha}_{\nu} - \frac{1}{4}\eta_{\mu\nu}F^2\right]$, the scalar operator $\mathrm{Tr}\left[F^2\right]$ and the pseudo-scalar operator $\mathrm{Tr}\left[F \star F\right]$. The holographic dictionary relates these operators to the fields of five dimensional non-critical string theory. In particular, the corresponding fields are the metric $g_{\mu\nu}$, the dilaton Φ and the axion[3] a. The two derivative effective string theory action that contains

[2]Examples of these type of theories, are QCD with a Banks-Zaks [71] fixed point, unparticle physics [72] and technicolor theories [12, 13, 15].

[3]Higher dimensional operators in QCD correspond to other string theory fields. The comprehensive discussion about the operators, the fields and the sourcing branes can be found in [22, 65]. These are not considered further in this thesis.

corresponding fields is the Einstein-Dilaton[4] action, which in the Einstein frame is

$$S_5 = -M_p^3 N_c^2 \int \mathrm{d}^5 x \sqrt{-g} \left[R - \frac{4}{3}(\partial\Phi) + V(\Phi) \right] + 2M_p^3 N_c^2 \int_\partial \mathrm{d}^4 x \sqrt{-h} K + S[a]. \tag{3.1}$$

Here K is the extrinsic curvature of the boundary. The effective Newton's constant is $G_5 = 1/(16\pi M_p^3 N^2)$ where N is related to the gauge d.o.f and M_p is the five-dimensional Planck scale.

For the Lorentz invariant YM vacuum, a five dimensional ansatz for the metric and the dilaton is

$$\mathrm{d}s^2 = b(z)^2 \left(\eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}z^2 \right), \quad \Phi(z) \propto \ln \lambda(z) \tag{3.2}$$

where the coordinates x^μ are identified with the 4D spacetime coordinates. The energy scale of the dual field theory is identified with the conformal factor $b(z)$ by a relation

$$E \equiv E_0 b(z). \tag{3.3}$$

Further, the coupling (λ_f) of the dual field theory is related to the dilaton as

$$\lambda_f \equiv \lambda = N_c e^\Phi. \tag{3.4}$$

An interesting combination of the above parameters is a beta function that is defined by the equation

$$\beta_f(\lambda_f) = \frac{\mathrm{d}\lambda_f}{\mathrm{d}\ln E}, \tag{3.5}$$

which tells how the coupling changes as a function of the energy scale. Similar function can be defined in the gravity side and is given by the equation

$$\beta(\lambda) = \frac{\mathrm{d}\lambda}{\mathrm{d}\ln b}. \tag{3.6}$$

Further, this is identified with (3.5), i.e., $\beta_f(\lambda_f) = \beta(\lambda)$.

The crucial part of the model is the non-trivial dilaton potential $V(\Phi) = V(\lambda)$, which encodes the dynamics of the theory. The potential in the small-λ limit is related to the UV geometry ($z \to 0$), whereas the large-λ regime is related to IR geometry. In addition, there is a one to one mapping between the beta function (3.5) and the dilaton potential. Thus, one can alternatively use the beta function to define the dynamics. In particular, the UV beta function can be mapped to the UV dilaton potential.

3.1.1 UV solution

The dual large-N gauge theory is assumed to be asymptotically free and conformally invariant in the UV. This implies that the UV geometry of the vacuum solution must be asymptotically Anti de Sitter. The pure five dimensional AdS solution (2.1) is found by taking $\Phi(z) = 0$ and $V(\Phi = 0) = 12/\mathcal{L}^2$ for which $b(z) = \mathcal{L}/z$. In IHQCD, the pure AdS is generalized to a asymptotically AdS space.

[4] The axion part of the action $S[a] \sim N_c$ does not change background solutions in the large-N_c limit [65] and is not considered further in this thesis.

The Einstein equations for the vacuum ansatz (3.2) are[5]

$$\begin{aligned} 6\frac{\dot{b}^2}{b^2}+3\frac{\ddot{b}}{b} &= b^2V(\Phi), \\ 6\frac{\dot{b}^2}{b^2}-3\frac{\ddot{b}}{b} &= \frac{4}{3}\dot{\Phi}^2. \end{aligned} \tag{3.7}$$

By defining a super-potential

$$W = -\frac{\dot{b}}{b^2}, \tag{3.8}$$

the above equations can be rewritten as first order equations

$$\begin{aligned} 12\frac{\dot{b}^2}{b^2}-3b\dot{W} &= b^2V(\Phi), \\ b\dot{W} &= \frac{4}{9}\dot{\Phi}^2. \end{aligned} \tag{3.9}$$

Further, the beta function (3.5) with identifications (3.3) and (3.4) is given by the equation

$$\beta(\lambda) = b\frac{\mathrm{d}\lambda}{\mathrm{d}b} \tag{3.10}$$

which leads to a relation

$$\dot{\lambda} = -Wb\beta(\lambda). \tag{3.11}$$

The reason for introducing the super-potential is that it gives an easy way to relate the beta function to the dilaton potential. This is accomplished by noting that

$$b\frac{\mathrm{d}W}{\mathrm{d}z} = \frac{4}{9}\frac{1}{\lambda^2}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}z}\right)^2 = -\frac{4}{9}\frac{1}{\lambda^2}Wb\beta(\lambda)\frac{\mathrm{d}\lambda}{\mathrm{d}z}, \tag{3.12}$$

$$\frac{\mathrm{d}W}{W} = -\frac{4}{9}\frac{\beta(\lambda)}{\lambda^2}\mathrm{d}\lambda, \tag{3.13}$$

from which one can integrate

$$W(\lambda) = W(0)\exp\left[-\frac{4}{9}\int \mathrm{d}\lambda\frac{\beta(\lambda)}{\lambda^2}\right]. \tag{3.14}$$

Finally, the relation between the beta function and the dilaton potential can be found by using the equation (3.9):

$$V(\lambda) = 12W(\lambda)^2\left[1-\left(\frac{\beta(\lambda)}{3\lambda}\right)^2\right]. \tag{3.15}$$

Thus, the potential can be fixed by specifying the beta function. Further, the beta function of IHQCD is identified with the beta function of the dual field theory with some minor

[5] The equation for the dilaton can be found combining these two. See [24].

differences that are discussed later. One can also identify λ with the corresponding 't Hooft coupling λ_f. In the UV, the dual field theory is taken to be asymptotically free ($\lambda_f \to 0$). This implies that the beta function at the UV is

$$\beta_f(\lambda_f) \equiv \beta(\lambda) = -\beta_0\lambda^2 - \beta_1\lambda^3 + \ldots, \quad \lambda \to 0. \tag{3.16}$$

where β_0 and β_1 are constants that depends on the details of the dual gauge theory. Using equations (3.14) and (3.15) one finds the small-λ (UV) expansion for the dilaton potential is

$$V(\lambda) = 12W(0)^2\left[1 + \frac{8}{9}\beta_0\lambda + \left(\frac{23}{81}\beta_0^2 + \frac{4}{9}\beta_1\right)\lambda^2 + \ldots\right]. \tag{3.17}$$

Comparing this with the pure five dimensional AdS space (2.1) fixes $W(0) = 1/\mathcal{L}$.

After specifying the dilaton potential (or the beta function) at the UV, one is ready to solve the UV asymptotics of $b(z)$ and $\lambda(z)$. Experience from the four dimensional large-N field theories suggest that the coupling has a logarithmic dependence on the energy scale ($\lambda \sim 1/\log E$). In the holographic picture, this would correspond to log-corrections in the AdS conformal factor, i.e., $E = E_0 b(z) = \mathcal{L}/z(1 + \mathcal{O}(\log z))$. Indeed, the UV solutions are found by studying equations

$$\dot{\lambda} \;=\; -Wb\beta(\lambda) \tag{3.18}$$

and

$$W = -\frac{\dot{b}}{b^2}. \tag{3.19}$$

The first non-trivial order leads to

$$\frac{\mathcal{L}}{-\beta_0\lambda^2}\mathrm{d}\lambda \;=\; -b\,\mathrm{d}z, \tag{3.20}$$

$$\frac{1}{\mathcal{L}}\left[1 + \frac{4}{9}\beta_0\lambda\right]\mathrm{d}z \;=\; -\frac{1}{b^2}\,\mathrm{d}b. \tag{3.21}$$

This pair of equations has a solution where the coupling is given by

$$\lambda = -\frac{1}{\beta_0 \log \Lambda z} \tag{3.22}$$

and the AdS conformal factor is

$$b(z) = \frac{\mathcal{L}}{z}\left[1 + \frac{4}{9}\frac{1}{\log \Lambda z}\right], \tag{3.23}$$

which has the expected form. The integration constant Λ is identified with the QCD scale Λ_{QCD}. Thus, IHQCD dynamically generates a mass scale in a way similar to large-N gauge theories. Higher order solutions can be found in [24].

3.1.1.1 Scheme dependence

The problem of scheme dependence is present in any attempts to solve the gauge field theory. Any physical observables must be scheme independent but different parametrizations of the coupling constant, or anything that is not scheme independent, leads to different descriptions. In IHQCD, the problem of various schemes is also present and is related to radial diffeomorphism. Possible schemes are reduced by picking up some specific frame for the metric. For example, in the conformal coordinate frame used above (3.2), the radial diffeomorphisms are reduced to the common scaling of the radial and boundary coordinates.

The scheme dependence of the field theory coupling can be understood as bulk field redefinitions. First, the relation between the bulk 't Hooft coupling λ and the field theory coupling λ_f is generally unknown[6]. This relation was studied in [22] where it was found that different sources of α' corrections have an effect on to the relation. In particular, in the UV

$$\lambda_f = \lambda\left(1 + c_1\lambda^2 + c_2\lambda^4 + \ldots\right). \tag{3.24}$$

Another definition that has similar vagueness as the coupling, is the relation between the field theory energy scale E and the conformal factor $b(z)$. In the UV, this relation is well understood but, in general, has corrections as one moves to smaller energies. In general, the relation can be written as a function of coupling

$$\frac{\mathrm{d}\log E}{\mathrm{d}\log b} = f(\lambda) = 1 + f_1\lambda^2 + f_2\lambda^4 + \ldots \tag{3.25}$$

where, to ensure monotonicity, one must have $f_i \geq 0$ and the constant term in is fixed by the standard AdS/CFT.

Next thing to ask is how the beta function is changed in the redefinitions. In field theory, the relation between the field theory coupling and energy was given by the beta function

$$\beta_f(\lambda_f) = \frac{\mathrm{d}\lambda_f}{\mathrm{d}\ln E} \tag{3.26}$$

which was identified with the bulk beta function $\beta(\lambda)$. This identification is clearly not true anymore. Instead, it is interesting that, even after the bulk redefinitions, the identification remains to be true up to first two terms in the UV expansion (3.16) [22]. In particular, this means that

$$\beta_f(\lambda_f) - \beta(\lambda) = 0 + \mathcal{O}(\lambda^4). \tag{3.27}$$

Also in the gauge theory side, the first two terms β_1 and β_2 in (3.16), are scheme independent. So in both sides of the duality the first two terms are scheme independent, which means that the gauge theory UV beta function can be used as an input for the UV dilaton potential.

Identifications between field theory and bulk variables in the IR regime can be very different from the UV identification done above. More comprehensive discussion about the scheme dependence can be found in [22, 23, 76].

[6] In the above one used $\lambda = \lambda_f$.

3.1.2 IR solutions

The IR dynamics of large-N field theories is generally far more demanding than the UV asymptotics which can be studied by using perturbation theory. In the IR, the coupling is large and some other method must be used. In IHQCD, the IR regime of the dual theory is identified with IR geometry where $\lambda \to \infty$.

In holographic models, a criterion for confinement is provided by the holographic Wilson loops [77, 78]. In field theory, the Wilson loop gives the potential energy between two static quarks and confinement appears when the potential energy increases linearly with distance, in which case the Wilson loop is said to obey an area law. A quantity related to the area law is the (QCD) string tension T_s.

Holographically, the Wilson loop is computed by using the Nambu-Goto action of a classical string, embedded in five dimensional space, with a rectangular loop with sides (space) L and (time) T on the AdS boundary [77, 78]. Then the Nambu-Goto action is proportional to the separation T and the energy between particles $E(L)$, i.e.,

$$TE(L) = S_{NG}[X^{\mu}_{min}(\sigma, \tau)]. \tag{3.28}$$

This calculation must be done in the string frame [77, 78]. In IHQCD, the relation between the Einstein frame and the string frame is

$$g^s_{\mu\nu} = e^{\frac{4}{3}\Phi} g_{\mu\nu} = \lambda^{\frac{4}{3}} g_{\mu\nu}. \tag{3.29}$$

The Nambu-Goto action for a classical string is

$$S_{NG} = T_f \int \mathrm{d}\sigma \mathrm{d}\tau \sqrt{-\det g^s} \tag{3.30}$$

where $g^s_{\alpha\beta}$ is the pull-back of the string frame metric to the string world volume and α, $\beta =$ 1, 2. The fundamental string tension is defined by the equation $T_f = 1/2\pi l_s^2$, where l_s is the fundamental string length.

The holographic Wilson loop calculation can be performed a follows. The end points of the string are fixed to the AdS boundary and the string falls down from the UV to the IR geometry. The separation between the endpoints of the string is given by [23]

$$L = 2 \int_0^{z_f} \mathrm{d}z \, \frac{1}{\sqrt{e^{4(A_s(z) - A_s(z_F))} - 1}}, \tag{3.31}$$

where z_f is a turning point where the string turns from the IR back to the UV boundary. The function $A_s(z)$ is a string frame scale factor, which is related to the Einstein frame scale factor by an equation

$$b(z)^2 e^{\frac{4}{3}\Phi} \quad = \quad e^{2(A(z) + \frac{2}{3}\Phi(z))} \equiv e^{2A_s(z)}. \tag{3.32}$$

To get area law for the potential energy $E(L)$ the separation between the endpoints of the string L must blow up at some point z. The potential infinity comes from the region near the turning point z_f which can be studied by expanding the integrand near the turning point z_f:

$$\frac{1}{\sqrt{4A_s'(z_f)(z_f - z) + 8A_s''(z_f)(z_f - z)^2 + \ldots}}. \tag{3.33}$$

This leads to finite L for generic z_f which cannot produce the area law, but grows indefinitely if there exists a stationary point $z_\star \approx z_f$ where $A'_s(z_\star) = 0$. The potential energy, in the limit of large separations, is given by [23]

$$E(L \to \infty) \sim T_f e^{2A_s(z_\star)} L, \tag{3.34}$$

and the confining (QCD) string tension is

$$T_s = T_f e^{2A_s(z_\star)}. \tag{3.35}$$

The outcome of the Wilson loop analysis is that the background geometry is confining if there exist a stationary point where

$$A'_s(z_\star) = A'(z_\star) + \frac{2}{3}\Phi'(z_\star) = 0. \tag{3.36}$$

Using the above criterion, one is ready to study which IR geometries give rise to the area law. In the below, class of spacetimes for which the conformal factor has an asymptotic form

$$b(z) = e^{A(z)} \to e^{-\left(\frac{z}{R}\right)^\alpha + \dots}, \quad \alpha > 0 \tag{3.37}$$

and the range of conformal coordinate is infinite $z \in (0, \infty)$, is studied.

Assuming asymptotically AdS space leads to the conclusion that the string frame scale factor behaves as

$$e^{A_S(z)} = b(z) e^{\frac{2}{3}\Phi} \to \frac{\mathcal{L}}{z} \frac{-1}{\beta_0 \log z\Lambda} \to \infty \tag{3.38}$$

as $z \to 0$. The geometry is confining with $z \in (0, \infty)$ if it satisfies the condition (3.36). This together with the UV asymptotics, implies that a necessary and sufficient condition for the geometry to confine is that $A_s(z)$ does not asymptote to $-\infty$ at the IR singularity $z \to \infty$ [22, 23].

To check for which α this condition is satisfied can use equation (3.7)

$$\begin{aligned} 6\frac{\dot{b}^2}{b^2} - 3\frac{\ddot{b}}{b} &= \frac{4}{3}\dot{\Phi}^2 \quad \Rightarrow \\ -\frac{9}{4}\left(\ddot{A} - \dot{A}^2\right) &= \dot{\Phi}^2 \end{aligned} \tag{3.39}$$

which for the background (3.37) has an asymptotic solution

$$\Phi \approx -\frac{3}{2}A(z) + \frac{3}{4}\log\left|\dot{A}(z)\right| + \Phi_0. \tag{3.40}$$

This equation, together with (3.36), solves (3.39) up to terms that are proportional to $(\ddot{A}/\dot{A})^2 \sim 1/z^2$. Further, the string frame scale factor in the IR is

$$A_s(z) \to \frac{1}{2}\log\left|\dot{A}(z)\right| \to \frac{(\alpha - 1)}{2}\log\left|\frac{z}{R}\right| \tag{3.41}$$

and the string frame metric behaves as

$$\mathrm{d}s_s^2 \sim \left(\frac{z}{R}\right)^{\alpha-1}\left(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}z^2\right). \tag{3.42}$$

The above results imply that a criterion for confinement for this type of geometries is simply $\alpha \geq 1$ for which $A_s(z)$ asymptotes to ∞ and has a minimum at some finite $z_\star$ leading to non-zero confining string tension T_s. Further, the IR behavior of the confining backgrounds implies that the dilaton and the beta function in the IR behaves as

$$
\begin{aligned}
V(\lambda) &= (\log \lambda)^{\frac{\alpha-1}{\alpha}} \lambda^{\frac{4}{3}} + \ldots, && (3.43) \\
\beta(\lambda) &= -\frac{3}{2}\lambda \left[1 + \frac{3}{4}\frac{\alpha-1}{\alpha}\frac{1}{\log \lambda} + \ldots\right], && (3.44)
\end{aligned}
$$

with $\alpha \geq 1$.

The above discussion can be generalized to geometries with different IR asymptotics and can be found in [23, 76].

3.1.3 Particle spectra

In confining gauge theories, the low energy particle spectrum is gapped. For QCD, this means that in the IR, one does not see massless gluons and quarks but, instead, one sees massive hadrons. For quarkless QCD, the low energy mass spectrum includes glueballs, which are bound states of the gluons. As IHQCD is thought to model large-N gauge theories that confine there should exist some way to calculate the masses of these particles using holography. The holographic calculation of the bound state masses can be done by considering fluctuations of fields around the background spacetime. Demanding these to satisfy specific boundary conditions, leads to a gapped and discrete particle spectrum [36]. The spins of the corresponding gauge theory particles are determined by studying how the fluctuations $\phi(z, x^i)$ transform under $SO(3)$ rotations.

A standard procedure for finding the field theory mass eigenstates is to write the fluctuations as

$$\phi(z, x) = \phi(z)\phi^{(4)}(x) \tag{3.45}$$

where $\phi^{(4)}(x)$ solves the four dimensional Klein-Gordon equation for free particle with a mass m:

$$\partial_i \partial^i \phi^{(4)}(x) = m^2 \phi^{(4)}(x). \tag{3.46}$$

The differential equations for $\phi(z)$ depends on m and the condition that the solution must be normalizable leads to a discrete set of possible four dimensional masses $m \equiv m_n$. These masses correspond the low energy excitations of the dual field theory.

In IHQCD, the background fields are Φ and $g_{\mu\nu}$. The spin-0 fluctuation is the gauge invariant (radial diffeomorphism) combination of the metric and dilaton whereas the spin-2 fluctuations are related to metric fluctuations δg_{ij} [79]. The equation for the scalar fluctuation is

$$\ddot{\phi}(z)_s + 3\frac{\dot{b}}{b}\dot{\phi}(z)_s - \left(\frac{\ddot{X}}{X} + 3\frac{\dot{b}}{b}\frac{\dot{X}}{X}\right)\phi(z)_s + m^2\phi(z)_s = 0, \tag{3.47}$$

where

$$X = \frac{\beta(\lambda(z))}{3\lambda(z)}. \tag{3.48}$$

For tensor fluctuations ($\phi(z)_t$) one simply removes the terms involving X [75]. The next step is to transform the equation to the form of a one dimensional Schrödinger equation. The transformation that takes it to the wanted form is

$$\psi(z) = \sqrt{b^3}\phi(z). \tag{3.49}$$

The Schrödinger equations for the fluctuations are then

$$-\ddot{\psi}_{T,S}(z) + V_{T,S}(z)\psi(z) = m^2\psi(z) \tag{3.50}$$

where

$$\begin{aligned} V_S(z) &= \frac{3}{2}\frac{\ddot{b}}{b} + \frac{3}{4}\frac{\dot{b}^2}{b^2} + \frac{\ddot{X}}{X} + 3\frac{\dot{b}}{b}\frac{\dot{X}}{X}, \\ V_T(z) &= \frac{3}{2}\frac{\ddot{b}}{b} + \frac{3}{4}\frac{\dot{b}^2}{b^2}. \end{aligned} \tag{3.51}$$

In the UV, the potential behaves as

$$V_{S,T}(z) = \frac{15}{4z^2} + \ldots, \quad z \to 0, \tag{3.52}$$

which leads to the UV solution

$$\psi_{T,S}(z) = C_1(m)z^{-3/2} + C_2(m)z^{5/2} \quad z \to 0. \tag{3.53}$$

A solution that corresponds to the mass eigenstates of the dual field theory must be normalizable. This implies that the parameter m must be chosen so that the solution is regular at the UV boundary, i.e., $C_1(m) = 0$. Further, to obtain a discrete spectrum this should be possible only for the discrete values of m. Experience with the Schrödinger potential suggests, that the potential should first decrease from its UV value and then again start to increase when approaching the IR singularity, forming a potential-well. This form for the potential leads to a discrete spectrum.

Let us consider the background for which the conformal factor goes as

$$b(z) = e^{A(z)} \to e^{-\left(\frac{z}{R}\right)^\alpha + \ldots}, \quad \alpha > 0 \tag{3.54}$$

which implies that the coupling has the form [23]

$$\lambda(z) = e^{\frac{3}{2}\left(\frac{z}{R}\right)^\alpha}\left(\frac{z}{R}\right)^{\frac{3}{4}(\alpha-1)}, \tag{3.55}$$

where R is some IR scale. The same form was found to be confining for $\alpha \geq 1$. In the IR, the potential (3.51) behaves as

$$V_{S,T}(z) \sim \frac{9}{4}R^{-2}\left(\frac{z}{R}\right)^{2(\alpha-1)}, \tag{3.56}$$

which is able to produce gapped and discrete states for $\alpha > 1$. For $\alpha = 1$, the spectrum is gapped but continuous after $m_{n_\star} \geq V_{S,T}(z \to \infty) = V^0_{S,T}$. It is interesting that the study of backgrounds that have gapped states leads to the same condition for α as the criteria for the backgrounds to confine. A more complete discussion on backgrounds and particle spectra can be found in [23, 79].

It is interesting to study the effects of the parameter α on the field theory particle spectrum [23]. For the lowest energy states, this must be done numerically but the higher modes can be analyzed by the WKB approximation. The quantization condition for eigenvalues m_n is approximately given by the quantization of the action integral

$$n\pi = \int_{z_1}^{z_2} \mathrm{d}z\,\sqrt{m_n^2 - V_{S,T}(z)} \tag{3.57}$$

where the turning points are approximately $z_1 = 0$ and $z_2 = R(Rm)^{\frac{1}{\alpha-1}}$. To probe very highly exited states, one takes $m_n^2 \gg V_{S,T}(z)$ for the intermediate region and fixes $V_{S,T}(z)$ to its asymptotic value. Now the integral can be calculated as:

$$n\pi \simeq m\int_0^{R(Rm)^{\frac{1}{\alpha-1}}} \mathrm{d}z\,\sqrt{1-\left[\frac{1}{mR}\left(\frac{z}{R}\right)^{\alpha-1}\right]^2} = (mR)^{\frac{\alpha}{\alpha-1}}\int_0^1 \mathrm{d}x\,\sqrt{1-x^2}. \tag{3.58}$$

From this, one obtains that the highly exited states behave as

$$m \sim n^{\frac{\alpha-1}{\alpha}}. \tag{3.59}$$

For QCD, the experiments and the lattice data suggest that $m^2 \sim n$, which is true for $\alpha = 2$.

3.1.4 Thermodynamics

The equilibrium thermodynamics of a system can be described by the partition function. The partition function can be calculated as a euclidean path integral on a manifold where the imaginary time is periodic with a period $\beta = 1/T$. In the large-N limit, the canonical partition function of the Einstein-Dilaton [24] model can be approximated by a sum over the saddle points:

$$Z(\beta) \approx e^{-S_1(\beta)} + e^{-S_2(\beta)} + \dots \tag{3.60}$$

where $S_i(\beta)$ are classical euclidean on-shell actions for the saddle-point solutions that share common asymptotics and, in particular, the same inverse temperature $\beta = 1/T$. The field theories of interest are assumed to have 3-dimensional rotational symmetry. In fact there are two types of euclidean solutions that share this symmetry. One is a thermal gas solution [24]

$$\mathrm{d}s^2 = b_0(z)^2\left(\mathrm{d}\tau^2 + \mathrm{d}x_j\mathrm{d}x^i + \mathrm{d}z^2\right), \quad \lambda_0(z) = Ne^{\Phi_0(z)}, \tag{3.61}$$

which is a euclidean version of the vacuum solution (3.2) with the inverse temperature β. This solution exists for all temperatures. Another finite temperature solution is given by a black hole solution

$$\mathrm{d}s^2 = b(z)^2\left(\frac{1}{f(z)}\mathrm{d}\tau^2 + \mathrm{d}x_j\mathrm{d}x^i + f(z)\mathrm{d}z^2\right), \quad \lambda(z) = Ne^{\Phi(z)}, \tag{3.62}$$

which has a horizon at $z = z_h$ where $f(z_h) = 0$. What makes this interesting is that there are different black hole solutions (with different temperatures) that can be characterized by the values of $\lambda(z_h) = \lambda_h$. The Einstein equations for the black hole ansatz are

$$6\frac{\dot{b}^2}{b^2} + 3\frac{\ddot{b}}{b} + 3\frac{\dot{b}}{b}\frac{\dot{f}}{f} = \frac{b^2}{f}V, \quad 6\frac{\dot{b}^2}{b^2} - 3\frac{\ddot{b}}{b} = \frac{4}{3}\dot{\Phi}^2 \tag{3.63}$$

$$\frac{\ddot{f}}{f} + 3\frac{\dot{b}}{b} = 0. \tag{3.64}$$

The thermal gas solution is related to the confining phase whereas the black hole solution is identified with the deconfined phase of the matter [24]. A transition between these phases, corresponds to a deconfinement phase transition in the dual field theory. To study the transition, one must first calculate the free energies of the corresponding solutions and find the solution which minimizes the free energy. Furthermore, the solution with the smallest free energy is identified as the dominating phase.

The free energy of the system i is identified with the euclidean on-shell action, i.e,

$$S(\beta) \equiv F_i = E_i - \beta^{-1} S_{\mathrm{e},\,i} \tag{3.65}$$

where E_i is the energy and $S_{\mathrm{e},\,i}$ is the entropy and $\beta^{-1} = T$ is the temperature of the system. For example, in the black hole solution these are related to the mass and the entropy of the black hole. All other thermodynamical quantities can be calculated from F. For example, pressure, entropy, specific heat and the speed of sound are given by the relations

$$p = -F, \quad S_{\mathrm{e}} = -\frac{\partial F}{\partial T}, \quad C_v = -T\frac{\partial^2 F}{\partial T^2}, \quad c_s^2 = \frac{S_{\mathrm{e}}}{C_v}. \tag{3.66}$$

The on-shell action for the holographic constructions is generally divergent near the AdS boundary and some renormalization produce for regulating the action must be used [24, 49, 50]. Another way to get around the divergences is to calculate the difference of the on-shell actions between the different phases. More precisely, the divergent part of the action is independent of the specific solution and thus, the difference between the phases leads to a finite answer, i.e, the divergent parts cancel each other [30].

The UV solution for the black hole ansatz is [76]

$$b(z) = b_0(z)\left[1 + \Omega\frac{z^4}{\mathcal{L}^3} + \cdots\right], \quad f(z) = 1 - \frac{C}{4}\frac{z^4}{\mathcal{L}^3} + \ldots, \tag{3.67}$$

$$\lambda(z) = \lambda(z)_0\left[1 + \frac{45}{8}\Omega\frac{z^4 \log \Lambda z}{\mathcal{L}^3} + \ldots\right], \tag{3.68}$$

where $b_0(z)$ and $f(z) = 1$ corresponds to the thermal gas phase [24]. The solutions differ at the $\mathcal{O}(z^4)$ and coefficients in the front of the terms can be related to the enthalpy (TS) and to the gluon condensate $\langle \mathrm{Tr}\,[F^2] \rangle$:

$$C = \frac{TS}{M_p^3 N^2 V_3}, \quad \Omega = \frac{22}{3(4\pi)^2}\frac{\langle \mathrm{Tr}\,[F^2]\rangle_{BH} - \langle \mathrm{Tr}\,[F^2]\rangle_0}{240 M_p^3 N^2}. \tag{3.69}$$

These quantities are related to the condition that the euclidean black hole metric must be regular at the horizon [24]. This relation can be easily found for the enthalpy C by integrating the equation (3.64), which gives

$$f(z) = 1 - C \int_0^z \mathrm{d}z' \, \frac{1}{b(z')^3}. \tag{3.70}$$

The derivative of $f(z)$ at the horizon z_h is

$$\dot{f}(z_h) = -C \frac{1}{b(z_h)^3}, \tag{3.71}$$

which can be related to the temperature and the entropy by noting that for the black hole [34, 35]

$$T = \frac{|\dot{f}(z_h)|}{4\pi}, \quad S = \frac{A_{\text{horizon}}}{4G_5} = \frac{b(z_h)^3 V_3}{4G_5}. \tag{3.72}$$

These results can be combined to give the enthalpy C. The gluon condensate Ω is similarly related to horizon quantities but cannot be expressed in a simple analytic form [24, 76].

The thermal gas phase exists for all temperatures and is confining for specific dilaton potentials. Thus, the experience in large-N field theories would suggest that the thermal gas phase dominates at low temperatures, thus, corresponding to the IR phase of the dual field theory.

The black-hole phase is more subtle. In fact, for dilaton potentials that lead to confinement, one finds that the solutions exist only for temperatures above some minimum $T_{\min}$. Further, these solutions have two (or more[7]) separate branches that are called the big and the small black hole. The different branches are related to the position of the horizon z_h. The different black hole branches can be also characterized by the value of the 't Hooft coupling on the horizon, i.e., $\lambda(z_h) = \lambda_h$. More precisely, the horizon of the big black hole with $\lambda_h \leq \lambda_h(T_{\min})$ is closer to the UV boundary while the horizon of the small black hole $\lambda_h > \lambda_h(T_{\min})$ is closer to the IR regime. In general, the temperature $T(\lambda_h)$ is a single valued function where as $\lambda_h(T)$ can take multiple values.

The difference between the free energies was calculated in [24] and it takes a simple form:

$$\frac{\triangle F}{M_p^3 N_c^2 V_3} = \frac{F_{BH} - F_0}{M_p^3 N_c^2 V_3} = 15\Omega - \frac{C}{4}. \tag{3.73}$$

As noted, for temperatures $0 < T < T_{\min}$ the only solution is the thermal gas. Thus, it is dominates the partition function. For temperatures $T > T_{\min}$ there is a competition between the free energies and there exists a critical temperature T_c after which the black hole phase dominates, i.e., $\triangle F(T > T_c \geq T_{\min}) < 0$. At the critical temperature, there is a phase transition from the thermal gas to the deconfined big black hole phase. The order of this transition depends on the specifics of the dilaton potential [24], but for confining backgrounds it is generally first order as is expected for large-N gauge theories .

In general, there can be more than two branches, and it is possible to have phase transitions between these branches. Black holes with more than two branches are considered in the following sections and in the articles II, III and IV.

[7] This is considered later in section 3.3.2.

3.2 Model for quarkless QCD

In this section, the model constructed in the articles [22, 23, 24] and its implications are studied. The construction motivates the choice of a dilaton potential, that is assumed to lead to the holographic dual for quarkless QCD, and specifies relations between the parameters of the Einstein-Dilaton system and quarkless QCD. After a short introduction, the results for thermodynamics, mass spectra and the transport properties are briefly discussed.

The basic properties of quarkless QCD that IHQCD is constructed to mimic, are asymptotic freedom at the UV and confinement at the IR. These properties are holographically encoded in to the specific choice of the dilaton potential. More precisely, the UV beta function has a scheme independent[8] form up to the first two terms

$$\beta(\lambda) = -\beta_0\lambda^2 - \beta_1\lambda^3 + \dots . \tag{3.74}$$

By using equations (3.14) and (3.15) this implies that the dilaton potential must behave as

$$V(\lambda) = \frac{12}{\mathcal{L}^2}\left[1 + \frac{8}{9}\beta_0\lambda + \left(\frac{23}{81}\beta_0^2 + \frac{4}{9}\beta_1\right)\lambda^2 + \dots\right], \quad \lambda \to 0. \tag{3.75}$$

Furthermore, the UV identifications of the five dimensional 't Hooft coupling (dilaton) and the beta function with the corresponding dual field theory coupling and beta function, i.e.,$\lambda = \lambda_t$ and $\beta(\lambda) = \beta_t(\lambda_t)$, imply that the coefficients of the UV beta function are fixed to be the same as for quarkless QCD [24], so that

$$\beta_0 = \frac{22}{3(4\pi)^2}, \quad \beta_1 = \frac{51}{121}\beta_0^2. \tag{3.76}$$

The confinement criterion was studied in subsection 3.1.2. Confinement in the dual field theory was found to imply that the dilaton potential must obey IR asymptotics of the form

$$V(\lambda) \sim (\log\lambda)^{\frac{\alpha-1}{\alpha}}\lambda^{\frac{4}{3}} \tag{3.77}$$

where[9] $\alpha \geq 1$. Furthermore, to get linear confinement ($m^2 \sim n$ for large n) one was led to fix $\alpha = 2$, which corresponds to the IR potential

$$V(\lambda) = \sqrt{\log\lambda}\,\lambda^{\frac{4}{3}} + \dots . \tag{3.78}$$

A dilaton potential that produces the above asymptotics is [24]

$$V(\lambda) = \frac{12}{\mathcal{L}^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\}, \tag{3.79}$$

where the coefficients V_0, V_1, V_2 are related to the UV beta function by identification

$$V_0 = \frac{8}{9}\beta_0,\; V_1\sqrt{V_2} = \left(\frac{23}{81}\beta_0^2 + \frac{4}{9}\beta_1\right), \tag{3.80}$$

[8] See 3.1.1.1.

[9] A more general discussion can be found in [23].

whereas V_3 is related to the IR behavior. After fixing the UV and IR asymptotics, as was done above, this potential has two free parameters, i.e., (V_1, V_3)[10] which can be used to fit the lattice data [76].

In addition to the parameters directly present in the dilaton potential, there are also more fundamental ones which are the relations between the gauge and the gravity parameters.

In the original AdS/CFT conjecture, the mapping between the fundamental parameters is clear and those are found by studying the D-brane solutions in different interpretations [30] that leads to the relations

$$g_{YM}^2 = 2\pi g_s, \quad \mathcal{L}^4 = 4\pi g_s l_s^4 N, \quad G_{10} = 8\pi^6(\alpha')^4 g_s^2. \tag{3.81}$$

In IHQCD and also in other phenomenological approaches, this mapping is more complicated. More precisely, to find the relations between the parameters, one needs to use some additional information which in general is a comparison between the calculations made on the both sides of the duality.

The fundamental parameters in IHQCD are

$$M_p, \ \mathcal{L}, \text{ and } \ l_s \tag{3.82}$$

where the first two are the 5D Planck mass and the AdS scale which are both directly seen in the IHQCD action (3.1). The last one is the fundamental string length, that appears, in the calculation of the confining (QCD) string tension (3.35). The 5D parameters M_p and $\mathcal{L}$ can be related to the QCD parameters by studying the entropy of the system at the UV regime, where QCD is asymptotically free. Entropy is calculated using equation (3.72), which in the UV gives

$$S = \frac{V_3}{4G_5}\frac{\mathcal{L}^3}{z_h^3}, \tag{3.83}$$

where the UV solution $b(z) = \mathcal{L}/z$ was used. In the UV, the relation between the black hole horizon and temperature is given by

$$T = \frac{1}{\pi z_h} \tag{3.84}$$

which, by using the relation $G_5 = 1/(16\pi M_p^3 N_c^2)$, leads to the UV entropy

$$S = 4\pi^4(\mathcal{L}M_p)^3 N_c^2 T^3 V_3. \tag{3.85}$$

On the other hand, calculating the d.o.f of quarkless QCD gives [24]

$$S_f = \frac{4}{45}\pi^2 N_c^2 T^3 V_3. \tag{3.86}$$

Now, since there is an assumed duality between these setups, these two expressions for the entropy should coincide which leads to the relation

$$(\mathcal{L}M_p)^3 = \frac{1}{45\pi^2}. \tag{3.87}$$

[10]This choice was done in [24]. One could also take the combination (V_2, V_3).

The fundamental string length l_s can be related to the confining string tension T_s. A holographic calculation for confining string tension was considered in the subsection 3.1.2 and the result was

$$T_s = T_f e^{2A_s(z_\star)} = T_f b_s(z_\star)^2 = T_f b(z_\star)^2 \lambda(z_\star)^{4/3} \tag{3.88}$$

where $z_\star$ is a point where $b_s'(z_\star) = 0$ and $T_f = \frac{1}{2\pi l_s^2}$. These equations lead to

$$\frac{l_s}{\mathcal{L}} = \frac{1}{\sqrt{2\pi T_f \mathcal{L}^2}} = \frac{b(z_\star)\lambda(z_\star)^{2/3}}{\sqrt{2\pi T_s \mathcal{L}^2}}. \tag{3.89}$$

The confining string tension cannot be related to any simple quantity at the UV, but instead can be compared with result found in lattice simulations. In particular, one can fix $l_s/\mathcal{L}$, by taking a lattice value of T_s and use Einstein equations to find a value for[11] $b(z_\star)\lambda(z_\star)^{2/3}$.

Interestingly, the ratio $l_s/\mathcal{L}$ is also related to the α' expansion and its value tells how well 5D string theory is approximated by the two derivative action (3.1) [76].

3.2.1 Thermodynamics

The thermodynamics of quarkless QCD can be studied on a lattice and the results indicate that some of the thermodynamical properties are almost identical for the different numbers of colors. In the reference [16] the thermodynamics of quarkless QCD were studied for $N = 3...8$. In particular, for quarkless QCD with $N \geq 3$ the thermodynamical quantities like pressure and energy density, when normalized to its corresponding UV value (asymptotically free theory), are almost independent of N. Furthermore, the phase structure seems to be very similar and, for example, the confined/deconfined transition is first order for all $N \geq 3$. These results indicate, that at least when the thermodynamics of the theory are studied, the quarkless QCD with $N = 3$ can be considered approximately as a large-N gauge theory. Furthermore, this leads to the conclusion that the holographic approach, which works only with large enough N, offers a new and a reliable way to study thermodynamical properties of the gauge theories.

In the references [16, 24] the authors compared the lattice data with the holographic calculations and found a very good fit. IHQCD contains only two free parameters that have effects on the shape of the thermodynamical quantities. These parameters are (V_1, V_3) and can be found from the dilaton potential

$$V(\lambda) = \frac{12}{\mathcal{L}^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\}. \tag{3.90}$$

The effects of the parameters can be summarized [76] as

1. V_1 controls how fast the thermodynamic quantities $p(T)/T^4$, $e(T)/T^4$ and $s(T)/T^3$ approach their asymptotically free values.

2. V_3 changes the latent heat per unit volume, which is related to the first order phase transition: $L_h = T\Delta s \simeq T_c s(T_c)$

[11] There are some subtleties in the value of $l_s/\mathcal{L}$ that are related to the identification of the field theory 't Hooft coupling with a dilaton $\lambda(z)$ [76].

According to the authors of articles [24, 76] the best fit to the lattice data is found with the values

$$V_1 = 14, \quad V_3 = 170. \tag{3.91}$$

For these values, the latent heat is

$$\frac{L_h}{N^2 T_c^4} = 0.31 \tag{3.92}$$

which matches with the lattice value for $N \to \infty$ and is slightly larger than the value for $N = 3$ where it is roughly $L_h/(3)^2 T_c^4 = 0.29$ [76].

An interesting thermodynamical quantity is the interaction measure also known as the trace anomaly:

$$\frac{e - 3p}{T^4}. \tag{3.93}$$

It is a dimensionless combination of the energy density and the pressure, and it roughly tells how far away the system is from conformality. For the original AdS/CFT conjecture, where the gauge theory is exactly conformal, the trace anomaly is always zero. Furthermore, in the case of IHQCD, where the dual field theory is taken to be quarkless QCD, it is generally non-zero and gets larger near the phase transition at the IR and vanishes at the UV.

After fixing the parameters (V_1, V_3) of the dilaton potential, the holographic calculation of the interaction measure agrees with the results found in the lattice simulation [16]. This is shown in Fig. 3.1.

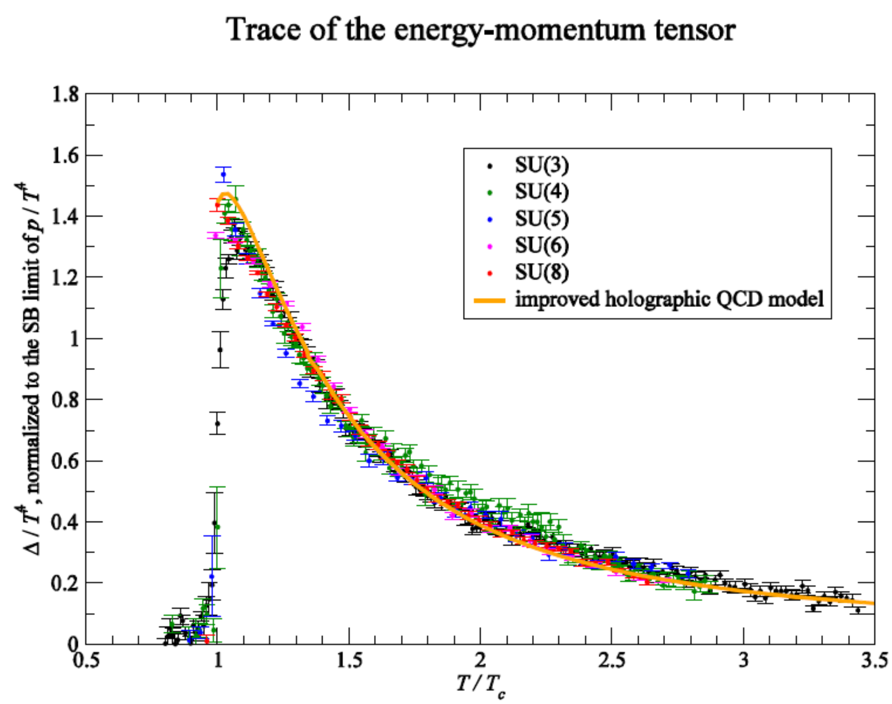

Figure 3.1: Lattice data [16] for the trace anomaly with $N = 3 \ldots 8$. The solid line is calculated using IHQCD with the parameters $V_1 = 14$, $V_3 = 170$. The value of the trace anomaly is normalized to p_{SB}/T^4, where p_{SB} is the Stefan-Boltzmann limit. Figure is taken from [16].

3.2.2 Mass spectra

The low energy spectrum of quarkless QCD contains various massive states. In this holographic construction one is able to study the glueballs with the quantum numbers $J^{PC} = 0^{++}, 0^{-+}, 2^{++}$. The operators related to these glueballs are Tr $[F^2]$, Tr $[F \star F]$ and the energy momentum tensor $T_{\mu\nu} = \mathrm{Tr}\left[F_{\mu\alpha}F^{\alpha}_{\nu} - \frac{1}{4}\eta_{\mu\nu}F^2\right]$. As explained in subsection 3.1.3, the holographic method for studying the dual field theory particle spectra is to consider the spectrum of normalizable fluctuations around the confining background. In IHQCD, the fluctuations corresponding to various glueballs are the fluctuations of the metric $\delta g_{\mu\nu}$, dilaton the $\delta\Phi$ and the axion[12] δa.

The fluctuations are specified by their properties under $SO(3)$ transformations rotations and parity P. In more detail, the 0^{-+} glueballs correspond to the fluctuation of axion field δa, the 2^{++} glueballs are related the tensor fluctuations of the metric δg_{ij} . The 0^{++} glueballs correspond to the combined fluctuations of the dilaton and the scalar part of the metric [79]:

$$\zeta = \psi - \frac{1}{3X(z)}\delta\Phi, \tag{3.94}$$

where $X(z) = \beta(\lambda(z))/3\lambda(z)$ and $\zeta(z)$ is constructed so that it is invariant under the radial diffeomorphism, i.e, it is gauge invariant.

The equation for the fluctuations is

$$-\ddot{\psi}_{T,S}(z) + V_{T,S}(z)\psi(z) = m^2\psi(z), \tag{3.95}$$

where S is related to 0^{++} and T is related to the tensor mode 2^{++}. Further, the Schrödinger potentials are

$$V_S(z) = \frac{3}{2}\frac{\ddot{b}}{b} + \frac{3}{4}\frac{\dot{b}^2}{b^2} + \frac{\ddot{X}}{X} + 3\frac{\dot{b}}{b}\frac{\dot{X}}{X}, \tag{3.96}$$

$$V_T(z) = \frac{3}{2}\frac{\ddot{b}}{b} + \frac{3}{4}\frac{\dot{b}^2}{b^2}, \tag{3.97}$$

where $X(z)$ and $b(z)$ are solutions to the background equations (3.7) with the dilaton potential (3.90) and the parameters (3.91). The glueball masses are found by studying the Schrödinger equation (3.95) and its normalizable solutions.

The mass spectra and the thermodynamical properties calculated using IHQCD can be compared with lattice data. This comparison is summarized in Table 3.1.

[12]The axion does not affect the background geometry in the limit where $N \to \infty$ and is not considered below. A discussion including the axion field can be found in [76].

	IHQCD	lattice $N = 3$	lattice $N \to \infty$	parameter
$p/(NT^4)\|_{T=2T_c}$	**1.2**	**1.2**	-	$V_1 = 14$
$L_h/(NT_c^4)$	**0.31**	0.28	**0.31**	$V_3 = 170$
$p/(NT^4)\|_{T=\infty}$	$\pi^2/45$	$\pi^2/45$	$\pi^2/45$	$(\mathcal{L}M_p)^3 = 1/45\pi^2$
$m_{0^{++}}/\sqrt{\sigma}$	**3.37**	3.56	**3.37**	$l_s/\mathcal{L} = 0.15$
$T_c/m_{0^{++}}$	0.167	-	0.177(7)	
$m_{2^{++}}/m_{0^{++}}$	1.36	1.40(4)	1.46(11)	

Table 3.1: The upper half of the table contains the quantities used as input (the boldface part) to IHQCD and the specific parameters in the 5D theory related to the fitting. In the lower part, the comparison with existing lattice data is made. The critical temperature T_c, in physical units, is 247 MeV. In the context of IHQCD, one is able to calculate the masses of exited glueballs. Further, comparing these with the lattice data indicates that the model gives a good global fit to quarkless QCD. This table is part of the table shown in [76] where similar comparisons related to the axion a is made.

3.3 Holographic Model for quasi-conformal theories

The first realization of the holographic principle was introduced by Maldacena in 1997. The conjectured duality between the four dimensional supersymmetric gauge theory and string theory on five dimensional AdS space was highly non-trivial and a great number of researchers started to study its possibilities. The original duality has been extended to different setups and some constructions are able to model non-supersymmetric QCD-like theories to high accuracy. One interesting class of theories that have been studied are quasi-conformal theories [12, 13, 15, 71, 72]. Quasi-conformal theories are nearly conformal at some energy scale and, thus, are more close to the original AdS/CFT than the highly non-conformal quarkless QCD studied in section 3.2.

Quasi-conformal theories and their holographic duals are the subjects of this section. Furthermore, the holographic model for the (walking-) technicolor theory is constructed and some results from the research articles at the end of this thesis are reviewed.

Technicolor theories are models for physics beyond the standard model, that address the electroweak symmetry breaking, the mechanism through which the elementary particles acquire masses. Early technicolor theories were modelled with scaled up quantum chromodynamics (QCD) which also inspired their name. A more detailed introduction to the subject can be found in [80].

Instead of introducing an elementary scalar particle (the Higgs boson) to explain the masses of fundamental particles, the technicolor models break the electroweak symmetry and generate masses for W and Z bosons through the dynamics of new gauge interactions between new particles called technigluons and technifermions. In particular, it is often assumed that the technicolor gauge group is SU(N_{TC}) under which the massless technifermions transform in the fundamental representation of the group. The gauge dynamics between the techniparticles dynamically generates a scale Λ_{TC} similar to QCD. Therefore, if the tecnifermions transform also under the electroweak gauge sector of the standard model this will lead to the electroweak symmetry breaking and to massive W and Z bosons.

Although technicolor is asymptotically free at very high energies, interactions between techniparticles must become strong and confining (and hence unobservable) at higher energies that have been experimentally probed. In particular, no tehcnihadrons have been seen in particle colliders. This dynamical approach to the electroweak symmetry breaking is natural and avoids the hierarchy problem of the standard model Higgs, but still has some other problems that are discussed below.

In order to produce quark and lepton masses, which in the standard model is done by Yukawa couplings, the technicolor has to be extended to larger gauge group which includes interactions between new gauge bosons and the standard model matter. The overall picture is that there is a large extended technicolor (ETC) gauge group in which technifermions, quarks, and leptons live in the same representations. Then the ETC is broken down to TC[13], and the standard model quarks and leptons emerge as the TC-singlet fermions. In more detail, the masses for quarks and leptons are generated by the interactions

$$\frac{1}{\Lambda_{\mathrm{ETC}}^2}\left\langle \bar{Q}_{\mathrm{TC}} Q_{\mathrm{TC}} \right\rangle_{\mathrm{ETC}} \bar{q}_{\mathrm{SM}} q_{\mathrm{SM}} \tag{3.98}$$

where Λ_{ETC} is the scale where the ETC is broken. The ETC gauge group where techniquarks and the standard model particles live in the same representation has its challenges including the experimental constraints on flavor-changing neutral currents due four Fermi interactions, i.e.,

$$\frac{1}{\Lambda_{\mathrm{ETC}}^2} \bar{q}_{\mathrm{SM}} q_{\mathrm{SM}} \bar{q}_{\mathrm{SM}} q_{\mathrm{SM}} \tag{3.99}$$

and the precision electroweak measurements [81]. Further, it is not known what is the extended technicolor dynamics.

In order to evade some of the above challenges in TC and ETC, much of the technicolor research focuses on exploring strongly-interacting gauge theories other than the scaled up versions of QCD . A particularly active framework is walking technicolor, which is able to enhance the masses for the quarks and leptons to observed values simultaneously keeping the ETC-scale large enough to avoid too large flavor-changing neutral currents and has also other advantages over the scaled up QCD [80].

To get a walking type of theory one needs to have a large number of techniquarks which is in contradiction to precision electroweak measurements [81]. This problem was recently solved by Sannino and Tuominen who proposed technicolor models with technifermions living in higher-dimensional representations of the technicolor gauge group [15, 80]. They argued that these, more "minimal" models, required fewer flavors of technifermions in order to exhibit walking behavior, making it easier to pass precision electroweak tests. For example, SU(3) gauge theory may exhibit walking with as few as two Dirac flavors of fermions in the adjoint or two-index symmetric representation. This is in contrast to a Banks-Zaks type of fixed point [71] where at least eight flavors of fermions in the fundamental representation of SU(3) are required to reach the near-conformal regime.

Whether walking can occur and lead to agreement with precision electroweak measurements is being studied through non-perturbative lattice simulations [82]. At the time of

[13] In general, the breaking of ETC down to TC may contain intermediate gauge groups.

writing the experiments at the Large Hadron Collider are expected to discover the mechanism responsible for electroweak symmetry breaking, which will be critical for determining whether the technicolor framework provides the correct description of nature.

3.3.1 Holographic model

Since the technifermions play an important role in the (walking-) technicolor theories, the holographic model should, in addition to the gauge degrees of freedom, contain also the fermionic degrees of freedom. In IHQCD, this is done by adding D4-$\bar{\text{D4}}$ branes to the gravity background. Furthermore, this introduces a new field to five dimensional action that is the tachyon field. This new field is mapped to the quark condensate via the holographic dictionary. The back reaction of the branes to the gravity background has been studied in the reference [70].

Instead of adding new fields (or D-branes) to the system, the model used in this section contains only the fields found also from the standard IHQCD. These fields are the metric $g_{\mu\nu}$ and the dilaton Φ. In this construction, the important effect of the fermions is directly fed into the form of the dilaton potential $V(\Phi)$. Thus, the analysis of the model stays identical to IHQCD but with a different dilaton potential.

In IHQCD, one used the properties of QCD (asymptotic freedom, confinement and the linear mass spectra) as an input to model, thus, these properties are also found from the holographic construction. In particular, the field theory input fixed a specific form for the dilaton potential at the UV and IR. In the holographic model of quasi-conformal theories (II, III and IV) one uses the beta function

$$\beta(\lambda) = -c\lambda^2 \frac{(1-\lambda)^2 + e}{1 + a\lambda^3} \tag{3.100}$$

as an input. This beta function has specific properties including asymptotic freedom at the UV, that correspond to the ETC scale, and quasi-conformality near $\lambda = 1$. Further, the conformality is approached for $e \to 0$ and for $e = 0$ the theory has a Banks-Zaks type of fixed point at $\lambda = 1$ [71]. The parameter e is identified with $e \sim N_{f,c} - N_f$, where $N_{f,c}$ denotes the critical number of flavors where the dual SU(N_{TC}) gauge theory develops an infrared stable fixed point. Using the input beta function and the equations (3.14), (3.15)

$$W(\lambda) = W(0) \exp\left[-\frac{4}{9}\int \mathrm{d}\lambda \frac{\beta(\lambda)}{\lambda^2}\right], \tag{3.101}$$

and

$$V(\lambda) = 12 W(\lambda)^2 \left[1 - \left(\frac{\beta(\lambda)}{3\lambda}\right)^2\right], \tag{3.102}$$

one is able to construct the corresponding dilaton potential. In technicolor theories, confinement at the IR introduces a scale (Λ_{TC}), which breaks the electroweak symmetry and, thus, one needs the potential to confine. The confining property was studied in section 3.1.2. The IR asymptotics of the confining potential must be of the form

$$V(\lambda) \sim (\log \lambda)^P \lambda^{2Q} \tag{3.103}$$

where[14]

$$P = \frac{\alpha - 1}{\alpha}, \quad Q = \frac{2}{3}, \quad \alpha \geq 1. \tag{3.104}$$

The input beta function (3.100) leads to the IR behavior

$$V(\lambda) \sim \lambda^{\frac{8c}{9a}}, \tag{3.105}$$

which by comparing with the confining property, fixes

$$\frac{c}{a} = \frac{3}{2}. \tag{3.106}$$

In addition, a logarithmic term in the IR potential (3.103) is needed. In the models considered in this thesis, the logarithmic dependence is simply added to the dilaton potential

$$V(\lambda) = V_0(\lambda) \sqrt{\frac{\log\,(F + \lambda^4)}{\log\, F}} \tag{3.107}$$

where $V_0(\lambda)$ is the potential (3.102) with $c/a = 3/2$ fixed. The logarithmic term[15] is constructed so that the UV behavior is unaffected and, furthermore, although there excited no clear (lattice) data for highly existed glueball spectrum to compare with, the spectra of type $m^2 \sim n$ similar to QCD is obtained with $\alpha = 2$. Here F is a parameter which sets the scale at which confinement effects kick in.

Since the quasi-conformal effects operate, by construction, at $\lambda = 1$ one certainly expect $F^{1/4} \gg 1$ in order to have a large separation between ETC (UV) and TC (IR) scales[16]. In particular, in this thesis one sets $F = 1000$.

Since the dilaton potential for quasi-conformal theory is now fixed, one can proceed similarly to the ordinary IHQCD and calculate various physical quantities including thermodynamics and the mass spectra.

3.3.2 Thermodynamics

In IHQCD, thermodynamics can be obtained by adding a black hole to the five dimensional background. The equations that must be solved are (3.63) and (3.64) which can only be done numerically. In section 3.1.4 it was discussed, that black-hole can have different branches. In particular, for the model of quarkless QCD there were two branches called the big and small black hole. The different branches are characterized by the sign of

$$\frac{\mathrm{d}\,T}{\mathrm{d}\,\lambda}. \tag{3.108}$$

For stable branches (phases) this derivative is negative, which in the ordinary IHQCD corresponded to the big black hole. Furthermore, there was a single first order transition from the

[14] A more general discussion is found in [23].
[15] In the article III authors used a different ansatz.
[16] This was needed so that on would not get too large flavor changing neutral currents.

big black hole phase to the vacuum phase, that corresponded to deconfiment/confinement transition.

The phase structure can be studied by calculating pressure which is given by

$$p = \frac{1}{4G_5}\int_{\lambda_h}^{\infty} \mathrm{d}\lambda_h \left(-\frac{\mathrm{d}T}{\mathrm{d}\lambda_h}\right) b^3(\lambda_h). \tag{3.109}$$

The thermodynamics of the quasi-conformal theory have not been studied on the lattice, but using the above holographic construction one is able to get some information on the phase structure. The analysis of the model done in III and IV, implies that are more than two black hole branches and one confining vacuum phase.

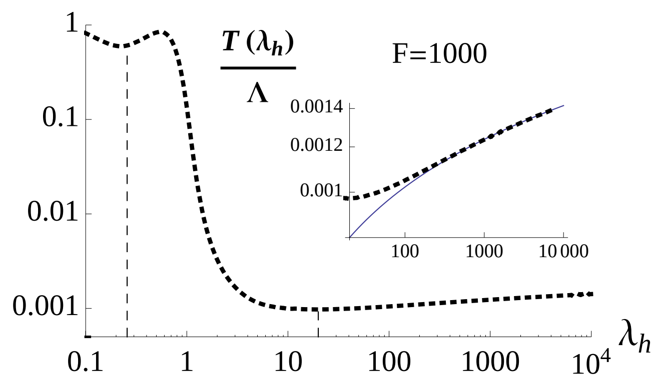

Figure 3.2: The function $T(\lambda_h)$ obtained for $e = 0.1$, $c = 13/(1+e)$, $F = 1000$. The first minimum is at $\lambda_h = 0.218$, $T/\Lambda = 0.8$ the second at $\lambda_h = 20$, $T/\Lambda = 0.001$. The corresponding transitions are at $T/\Lambda = 0.653$ and at $T/\Lambda = 0.00098$. The behavior at large λ_h (inset) can be fitted by $0.00050 \log^{0.47} \lambda_h$.

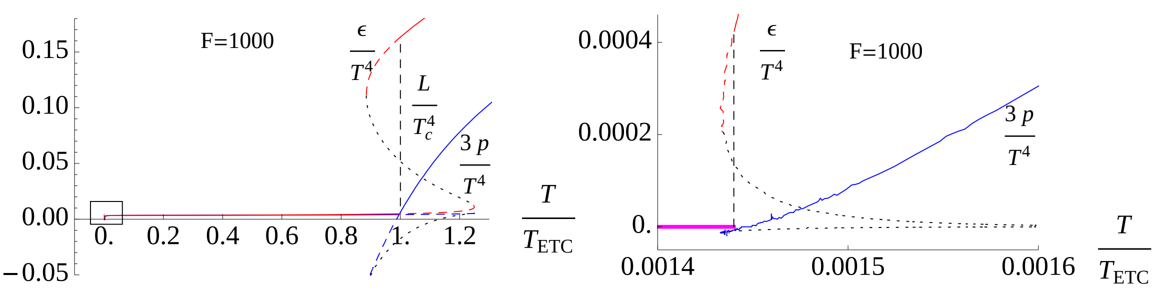

Figure 3.3: Left: The equation of state obtained for $e = 0.1$, $c = 13/(1+e)$, $F = 1000$ for the region around the first order ETC phase transition at $T_{\mathrm{ETC}} = 0.602\Lambda$. Dotted lines are unstable, dashed lines metastable (supercooled or -heated). Right: The boxed confinement transition to the low T phase with $p = 0$ at $T_{\mathrm{TC}} = 0.000977\Lambda$. Dotted lines are unstable, dashed lines metastable (supercooled or -heated).

The black hole branches are shown in Fig. 3.2 [75] and the corresponding equation of the state calculated from (3.109) is shown in Fig. 3.3 [75]. Now λ_h is the value of 't Hooft coupling at the horizon, which characterizes the different solutions to the equations (3.63) and (3.64). Fig. 3.2 shows that there is a new structure at $\lambda_h \lesssim 1$ that corresponds to a new black hole branch. For $\lambda_h \gtrsim 0.218$ the derivative $\mathrm{d}\,T/\mathrm{d}\,\lambda$ is positive and, thus, the black hole branch is unstable. For some critical $\lambda_h \lesssim 0.218$ there is a phase transition from the UV (big) black hole branch to another stable black hole branch which corresponds to temperature $T_{\mathrm{ETC}} = 0.602\Lambda$ and $\lambda_h \gtrsim 1$. This transition can be thought to correspond to the ETC breaking where the asymptotically free theory gets to the quasi-conformal phase. For some critical $\lambda_h \lesssim 20$, the pressure (3.109) becomes negative and the vacuum phase starts to dominate. Further, this is identified as a deconfiment/confinement transition that corresponds to TC transition with $T_{\mathrm{TC}} = 0.000977\Lambda$. For $e = 0$ the TC transition is absent and the quasi-conformal phase continuous down to zero temperature, so the black hole phase dominates all the way to $T = 0$.

It will be interesting to see, whether the phase structure found in this thesis will coincide with lattice results to be found in future studies. In this dual construction, the fermion degrees of freedom were assumed to have an effect only to the form of the dilaton potential which is clearly not a valid approximation. There is a recent study [70] where the holographic construction lies on more solid ground. This model can be used to study thermodynamics in more detail and check the validity of the method used in this thesis.

3.3.3 Mass spectra and quasi-normal modes

The mass spectrum can be found similarly as for IHQCD, but with the dilaton potential corresponding to quasi-conformal theories (3.107). The equations for the gauge-invariant scalar and the tensor fluctuations can be found in section 3.2.2. Note, that in those equations the beta function is not (3.100) but must be calculated numerically from the dilaton potential (3.107), which leads to logarithmic corrections in the input beta function (3.100). The result of this analysis can be found in IV and the findings are reviewed in the Fig. 3.4.

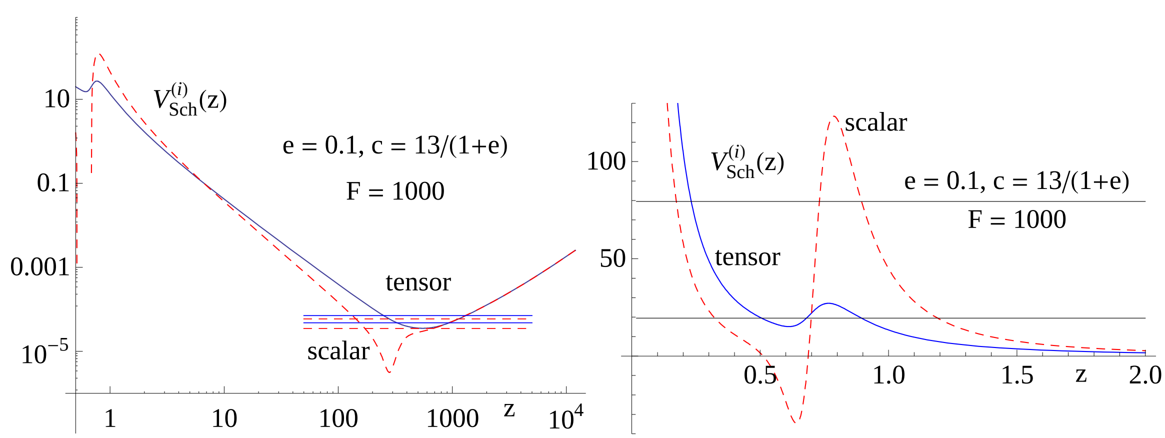

Figure 3.4: Left: The scalar (dashed line) and tensor (continuous line) potentials in the IR large-z TC region for $e = 0.1$, $c = 13/(1+e)$, $F = 1000$ (note the logarithmic scale). The two lowest scalar and tensor excitations are plotted and the ordering is $E_{S,0} < E_{T,0} < E_{S,1} < E_{T,1}$. Right: The scalar and tensor potentials in the ETC (UV, small-z) region (note linear scale). For $z \to 0$ both potentials approach $\sim 15/\left(4z^2\right)$.

The Schrödinger potential for both modes has similar structure which can be found in the Fig. 3.4, where also the lowest energy levels are shown. These are the glueballs related to the confining TC phase.

There is also a promising looking dip in the potentials near the ETC scale, which in principle could confine (Fig. 3.4). In fact, it was found that the dip can only produce metastable states and these states have tunneling actions of the order of $\mathcal{O}(1)$ (article IV). Furthermore, the limit $e \to 0$ was considered and its effect to the TC glueball mass was found to reproduce a Miransky scaling [83]

$$m \sim \exp\left(-\frac{D}{\sqrt{e}}\right), \quad D = \left(\frac{2}{3} + \frac{1}{c}\right)\pi. \tag{3.110}$$

The quasi-normal modes which are the finite temperature counterparts for glueballs, have non-zero imaginary parts. These were also studied in the research article IV. As expected, it was found that the imaginary part of the quasi-normal mode in the TC regime vanishes when the temperature is taken to zero. Another interesting aspect is that near the ETC scale, as can be seen from the Fig. 3.5, the peak in the potential "melts" away when the temperature is increased. This implies that even the lowest metastable states of the ETC region are almost entirely imaginary.

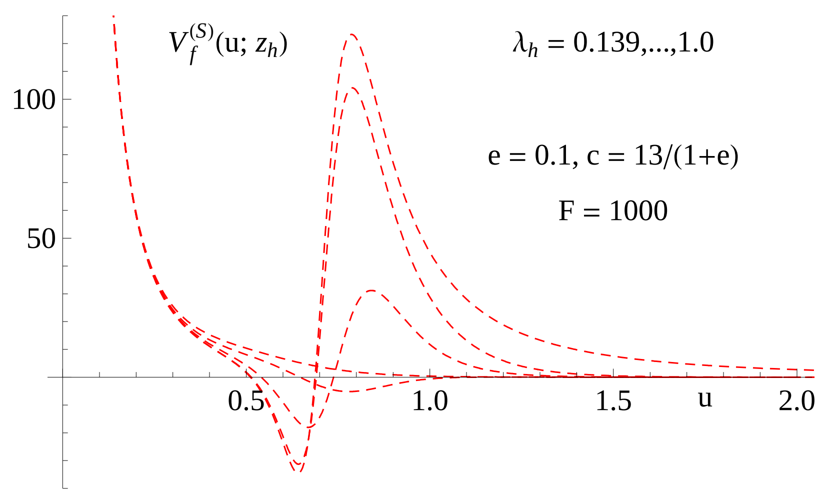

Figure 3.5: The scalar potential $V_f^{(S)}(u, z_h)$ for $\lambda_h = 0.138, 0.329, 0.520, 0.711, 1.0$, $z_h = 0.57, 0.719, 0.724, 0.92, 2.96$, $\pi T \approx 1/z_h$. The peak disappears when approaching the UV regime (when λ_h decreases).

Chapter 4

Summary

In this thesis, the AdS/CFT conjecture and its generalizations were studied. First, the basic blocks of the duality were introduced and some steps that lead to the conjecture were discussed. Next generalizations of the duality were studied and dual models for some IR phenomena in field theory were considered. After the general discussion, the specific model used in the thesis, the Improved Holographic QCD (IHQCD), was introduced and its application to quarkless QCD was studied. It was found that IHQCD is able to fit the thermodynamics of quarkless QCD computed with lattice simulations to very high accuracy. In addition, the holographic construction was able to produce roughly the correct masses for QCD glueballs. The IHQCD model used in this thesis can also be used to study the transport properties of a strongly coupled plasma, which makes it a very interesting theoretical laboratory to study the quark-gluon plasma seen at RHIC and LHC [67, 84]. Finally, IHQCD can be extended to model condensed matter physics as was done in [85].

After a short introduction to technicolor theories, the corresponding holographic model was constructed. The construction did not contain fermions as explicit degrees of freedom but, instead, the dilaton potential of ordinary IHQCD was modified to reflect their dynamics (II, III and IV). It was done so that it reproduced the expected structure for the theory, namely a new quasi-conformal regime at intermediate energies. Next, the phase structure of the dual quasi-conformal theories was reviewed. It was found that it had tree distinct phases, free ETC gluons at large temperatures, a quasi-conformal phase and the confining phase at low temperatures. In addition, mass spectra and quasi-normal modes were considered. Thermodynamics of technicolor theories has not been studied on the lattice, so it remains to be seen whether the phase structure of concrete field theory will look similar to what was found in this work.

In the (walking-) technicolor theories the fermionic degrees of freedom play a crucial role and, thus, should be included in the holographic model too. This can be done by introducing flavor branes to the system that modifies the background geometry. The vacuum of this theory have recently been studied by Järvinen and Kiritsis [70], in which they found the Miransky scaling that indicates that the method used in this thesis shares some properties with the setup including dynamical fermions. Their construction can be straightforwardly generalized to finite temperatures and this remains to be done.

Bibliography

[1] R. Gupta, “Introduction to lattice QCD: Course,” hep-lat/9807028.

[2] S. Scherer, “Introduction to chiral perturbation theory,” Adv. Nucl. Phys. **27**, 277 (2003) [hep-ph/0210398].

[3] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [hep-th/9711200].

[4] L. Susskind, “Dual symmetric theory of hadrons. 1.,” Nuovo Cim. A **69**, 457 (1970).

[5] B. Andersson, G. Gustafson, G. Ingelman and T. Sjostrand, “Parton Fragmentation and String Dynamics,” Phys. Rept. **97**, 31 (1983).

[6] A. C. Irving and R. P. Worden, “Regge Phenomenology,” Phys. Rept. **34**, 117 (1977).

[7] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].

[8] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from noncritical string theory,” Phys. Lett. B **428**, 105 (1998) [hep-th/9802109].

[9] G. ’t Hooft, “Dimensional reduction in quantum gravity,” gr-qc/9310026.

[10] L. Susskind, “The World as a hologram,” J. Math. Phys. **36**, 6377 (1995) [hep-th/9409089].

[11] G. ’t Hooft, “A Planar Diagram Theory for Strong Interactions,” Nucl. Phys. B **72**, 461 (1974).

[12] S. Weinberg, “Implications of Dynamical Symmetry Breaking,” Phys. Rev. D **13**, 974 (1976).

[13] L. Susskind, “Dynamics of Spontaneous Symmetry Breaking in the Weinberg-Salam Theory,” Phys. Rev. D **20**, 2619 (1979).

[14] C. T. Hill and E. H. Simmons, “Strong dynamics and electroweak symmetry breaking,” Phys. Rept. **381**, 235 (2003) [Erratum-ibid. **390**, 553 (2004)] [hep-ph/0203079].

[15] F. Sannino and K. Tuominen, "Orientifold theory dynamics and symmetry breaking," Phys. Rev. D **71**, 051901 (2005) [hep-ph/0405209].

[16] M. Panero, "Thermodynamics of the QCD plasma and the large-N limit," Phys. Rev. Lett. **103**, 232001 (2009) [arXiv:0907.3719 [hep-lat]].

[17] M. Gyulassy, "The QGP discovered at RHIC," nucl-th/0403032.

[18] J. Polchinski and M. J. Strassler, "Hard scattering and gauge / string duality," Phys. Rev. Lett. **88**, 031601 (2002) [arXiv:hep-th/0109174].

[19] J. Erdmenger, N. Evans, I. Kirsch and E. Threlfall, "Mesons in Gauge/Gravity Duals - A Review," Eur. Phys. J. A **35**, 81 (2008) [arXiv:0711.4467 [hep-th]].

[20] T. Sakai and S. Sugimoto, "Low energy hadron physics in holographic QCD," Prog. Theor. Phys. **113**, 843 (2005) [hep-th/0412141].

[21] T. Sakai and S. Sugimoto, "More on a holographic dual of QCD," Prog. Theor. Phys. **114**, 1083 (2005) [hep-th/0507073].

[22] U. Gursoy and E. Kiritsis, "Exploring improved holographic theories for QCD: Part I," JHEP **0802**, 032 (2008) [arXiv:0707.1324 [hep-th]].

[23] U. Gursoy, E. Kiritsis and F. Nitti, "Exploring improved holographic theories for QCD: Part II," JHEP **0802**, 019 (2008) [arXiv:0707.1349 [hep-th]].

[24] U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti, "Holography and Thermodynamics of 5D Dilaton-gravity," JHEP **0905**, 033 (2009) [arXiv:0812.0792 [hep-th]].

[25] S. A. Hartnoll, "Horizons, holography and condensed matter," arXiv:1106.4324 [hep-th].

[26] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. **323**, 183 (2000) [hep-th/9905111].

[27] E. D'Hoker and D. Z. Freedman, "Supersymmetric gauge theories and the AdS / CFT correspondence," hep-th/0201253.

[28] C. V. Johnson, "D-brane primer," hep-th/0007170.

[29] I. R. Klebanov, "TASI lectures: Introduction to the AdS / CFT correspondence," hep-th/0009139.

[30] J. L. Petersen, "Introduction to the Maldacena conjecture on AdS / CFT," Int. J. Mod. Phys. A **14**, 3597 (1999) [hep-th/9902131].

[31] P. H. Ginsparg, "Applied Conformal Field Theory," hep-th/9108028.

[32] S. W. Hawking, "Black hole explosions," Nature **248**, 30 (1974).

[33] S. W. Hawking, "Particle Creation by Black Holes," Commun. Math. Phys. **43**, 199 (1975) [Erratum-ibid. **46**, 206 (1976)].

[34] J. D. Bekenstein, "Black holes and entropy," Phys. Rev. D **7**, 2333 (1973).

[35] G. W. Gibbons and S. W. Hawking, "Action Integrals and Partition Functions in Quantum Gravity," Phys. Rev. D **15**, 2752 (1977).

[36] E. Witten, "Anti-de Sitter space, thermal phase transition, and confinement in gauge theories," Adv. Theor. Math. Phys. **2**, 505 (1998) [hep-th/9803131].

[37] S. Weinberg, "The quantum theory of fields. Vol. 3: Supersymmetry," Cambridge, UK: Univ. Pr. (2000) 419 p

[38] J. Wess and J. Bagger, "Supersymmetry and supergravity," Princeton, USA: Univ. Pr. (1992) 259 p

[39] J. Polchinski, "String theory. Vol. 1: An introduction to the bosonic string," Cambridge, UK: Univ. Pr. (1998) 402 p

[40] J. Polchinski, "String theory. Vol. 2: Superstring theory and beyond," Cambridge, UK: Univ. Pr. (1998) 531 p

[41] J. Scherk and J. H. Schwarz, "Dual Models For Nonhadrons," Nucl. Phys. B **81**, 118 (1974).

[42] J. Polchinski, "Dirichlet Branes and Ramond-Ramond charges," Phys. Rev. Lett. **75**, 4724 (1995) [hep-th/9510017].

[43] K. S. Stelle, "BPS branes in supergravity," In *Trieste 1997, High energy physics and cosmology* 29-127 [hep-th/9803116].

[44] S. S. Gubser, I. R. Klebanov and A. W. Peet, "Entropy and temperature of black 3-branes," Phys. Rev. D **54**, 3915 (1996) [hep-th/9602135].

[45] R. C. Myers, "Dielectric branes," JHEP **9912**, 022 (1999) [hep-th/9910053].

[46] I. R. Klebanov, "World volume approach to absorption by nondilatonic branes," Nucl. Phys. B **496**, 231 (1997) [hep-th/9702076].

[47] S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, "String theory and classical absorption by three-branes," Nucl. Phys. B **499**, 217 (1997) [hep-th/9703040].

[48] N. Berkovits and C. Vafa, "Towards a Worldsheet Derivation of the Maldacena Conjecture," JHEP **0803**, 031 (2008) [AIP Conf. Proc. **1031**, 21 (2008)] [arXiv:0711.1799 [hep-th]].

[49] K. Skenderis, "Lecture notes on holographic renormalization," Class. Quant. Grav. **19**, 5849 (2002) [hep-th/0209067].

[50] V. Balasubramanian and P. Kraus, "A Stress tensor for Anti-de Sitter gravity," Commun. Math. Phys. **208**, 413 (1999) [hep-th/9902121].

[51] R. B. Mann and D. Marolf, "Holographic renormalization of asymptotically flat spacetimes," Class. Quant. Grav. **23**, 2927 (2006) [hep-th/0511096].

[52] E. Imeroni, "The Gauge / string correspondence towards realistic gauge theories," hep-th/0312070.

[53] J. Polchinski and M. J. Strassler, "The String dual of a confining four-dimensional gauge theory," hep-th/0003136.

[54] O. Aharony, "The NonAdS / nonCFT correspondence, or three different paths to QCD," hep-th/0212193.

[55] J. Babington, D. E. Crooks and N. J. Evans, "A Nonsupersymmetric deformation of the AdS / CFT correspondence," JHEP **0302**, 024 (2003) [hep-th/0207076].

[56] L. Da Rold and A. Pomarol, "Chiral symmetry breaking from five dimensional spaces," Nucl. Phys. B **721**, 79 (2005) [hep-ph/0501218].

[57] D. T. Son and M. A. Stephanov, "QCD and dimensional deconstruction," Phys. Rev. D **69**, 065020 (2004) [hep-ph/0304182].

[58] U. Gursoy, "Deconfinement and Thermodynamics in 5D Holographic Models of QCD," Mod. Phys. Lett. A **23**, 3349 (2009) [arXiv:0904.2750 [hep-th]].

[59] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, "Gauge/String Duality, Hot QCD and Heavy Ion Collisions," arXiv:1101.0618 [hep-th].

[60] A. Karch and E. Katz, "Adding flavor to AdS / CFT," JHEP **0206**, 043 (2002) [hep-th/0205236].

[61] M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, "Meson spectroscopy in AdS / CFT with flavor," JHEP **0307**, 049 (2003) [hep-th/0304032].

[62] I. Kirsch and D. Vaman, "The D3 / D7 background and flavor dependence of Regge trajectories," Phys. Rev. D **72**, 026007 (2005) [hep-th/0505164].

[63] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, "QCD and a holographic model of hadrons," Phys. Rev. Lett. **95**, 261602(2005) [hep-ph/0501128].

[64] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, "Linear confinement and AdS/QCD," Phys. Rev. D **74**, 015005 (2006) [hep-ph/0602229].

[65] E. Kiritsis, "Dissecting the string theory dual of QCD," Fortsch. Phys. **57**, 396 (2009) [arXiv:0901.1772 [hep-th]].

[66] U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti, “Improved Holographic Yang-Mills at Finite Temperature: Comparison with Data,” Nucl. Phys. B **820**, 148 (2009) [arXiv:0903.2859 [hep-th]].

[67] U. Gursoy, E. Kiritsis, G. Michalogiorgakis and F. Nitti, “Thermal Transport and Drag Force in Improved Holographic QCD,” JHEP **0912**, 056 (2009) [arXiv:0906.1890 [hep-ph]].

[68] I. Iatrakis, E. Kiritsis and A. Paredes, “An AdS/QCD model from Sen’s tachyon action,” Phys. Rev. D **81**, 115004 (2010) [arXiv:1003.2377 [hep-ph]].

[69] I. Iatrakis, E. Kiritsis and A. Paredes, “An AdS/QCD model from tachyon condensation: II,” JHEP **1011**, 123 (2010) [arXiv:1010.1364 [hep-ph]].

[70] M. Jarvinen and E. Kiritsis, “Holographic Models for QCD in the Veneziano Limit,” arXiv:1112.1261 [hep-ph].

[71] T. Banks and A. Zaks, “On the Phase Structure of Vector-Like Gauge Theories with Massless Fermions,” Nucl. Phys. B **196**, 189 (1982).

[72] H. Georgi, “Unparticle physics,” Phys. Rev. Lett. **98**, 221601 (2007) [hep-ph/0703260].

[73] J. Alanen and K. Kajantie, “Thermodynamics of a field theory with infrared fixed point from gauge/gravity duality,” Phys. Rev. D **81**, 046003 (2010) [arXiv:0912.4128 [hep-ph]].

[74] J. Alanen, K. Kajantie and K. Tuominen, “Thermodynamics of Quasi Conformal Theories From Gauge/Gravity Duality,” Phys. Rev. D **82**, 055024 (2010) [arXiv:1003.5499 [hep-ph]].

[75] J. Alanen, T. Alho, K. Kajantie and K. Tuominen, “Mass spectrum and thermodynamics of quasi-conformal gauge theories from gauge/gravity duality,” Phys. Rev. D **84**, 086007 (2011) [arXiv:1107.3362 [hep-th]].

[76] U. Gursoy, E. Kiritsis, L. Mazzanti, G. Michalogiorgakis and F. Nitti, “Improved Holographic QCD,” Lect. Notes Phys. **828**, 79 (2011) [arXiv:1006.5461 [hep-th]].

[77] J. M. Maldacena, “Wilson loops in large N field theories,” Phys. Rev. Lett. **80**, 4859 (1998) [hep-th/9803002].

[78] S. -J. Rey and J. -T. Yee, “Macroscopic strings as heavy quarks in large N gauge theory and anti-de Sitter supergravity,” Eur. Phys. J. C **22**, 379 (2001) [hep-th/9803001].

[79] E. Kiritsis and F. Nitti, “On massless 4D gravitons from asymptotically AdS(5) space-times,” Nucl. Phys. B **772**, 67 (2007) [hep-th/0611344].

[80] F. Sannino, “Conformal Dynamics for TeV Physics and Cosmology,” Acta Phys. Polon. B **40**, 3533 (2009) [arXiv:0911.0931 [hep-ph]].

[81] M. E. Peskin and T. Takeuchi, "A New constraint on a strongly interacting Higgs sector," Phys. Rev. Lett. **65**, 964 (1990).

[82] T. Appelquist, A. Avakian, R. Babich, R. C. Brower, M. Cheng, M. A. Clark, S. D. Cohen and G. T. Fleming *et al.*, "Toward TeV Conformality," Phys. Rev. Lett. **104**, 071601 (2010) [arXiv:0910.2224 [hep-ph]].

[83] D. B. Kaplan, J. -W. Lee, D. T. Son and M. A. Stephanov, "Conformality Lost," Phys. Rev. D **80**, 125005 (2009) [arXiv:0905.4752 [hep-th]].

[84] G. Policastro, D. T. Son and A. O. Starinets, "The Shear viscosity of strongly coupled N=4 supersymmetric Yang-Mills plasma," Phys. Rev. Lett. **87**, 081601 (2001) [hep-th/0104066].

[85] C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, "Effective Holographic Theories for low-temperature condensed matter systems," JHEP **1011**, 151 (2010) [arXiv:1005.4690 [hep-th]].